



ISSN: 1813-162X (Print) ; 2312-7589 (Online)

Tikrit Journal of Engineering Sciences

available online at: <http://www.tj-es.com>
**TJES**  
 Tikrit Journal of  
 Engineering Sciences

Salim Y. Kasim

Raaid M. Hameed \*

 Mechanical Engineering Department  
 College of Engineering  
 Tikrit University  
 Salahaldeen  
 Iraq

# A Study of Two- Dimensional Non Linear Model of Truck Vehicle and its Impact on Determining the Maximum Speed of Driver Comfort

## ABSTRACT

Ride comfort is one of the important requirement which cars manufactures is try to achieved it because its directly effect on the sales rate, so it was to be tacking the factors that effect on the comfort degree of car users , in this field it has been concentrating on the driver comfort because of his responsibility on the car drive and on passengers safety as well as to focus on the road and see the obstacles, this is achieved by isolating the vehicle body from the vibrations results due to irregularly by use suitable suspension system . In order to obtain accurate results, we study the non - linear disposition of system parts by taking into account the most prominent source of non-linearity represented by large angle of swing and non-linearity of spring and damper behavior, appropriate model was chosen by taking non-linear half truck vehicle model with four degree of freedom and by considering rigid vehicle body using two types of excitation half sin wave and trapezoidal obstacles as an expression of road irregularly. A Fortran computer program for simulation the results represented by displacement, velocity and acceleration in the center of gravity of vehicle body (C.G) , then comparison with recently published research was doing and it shown an acceptable compliance degree, to find out the improvement extent when taking into account the system non - linearity we compare with linear halve truck vehicle model and found that the adopting of non – linearity leads to improve the results, then an investigation to the driver comfort subject been carried out to know what we have achieved in this field by choose Ride Comfort Level (RCL) as comfortable criteria then the results have been used to determine the maximum speed that keeps the vehicle within comfort limit using three types of obstacles . It was found that the adopting of non-linearity in the accounts gives more speed range. Thus it can be say that we have achieved the goal of this research.

### Keywords:

 Non-linearity  
 half vehicle  
 4 DOF  
 ride comfort level  
 suspension systems

## ARTICLE INFO

### Article history:

 Received 04 October 2017  
 Accepted 16 January 2018  
 Available online 17 June 2018

© 2018 TJES, College of Engineering, Tikrit University

DOI: <http://dx.doi.org/10.25130/tjes.25.2.04>

## دراسة نموذج ثنائي البعد لا خطي لمركبة حمل وتأثيره على تحديد السرعة القصوى لراحة السائق

### الخلاصة

راحة الراكب هي احد المطالب المهمة التي تسعى لتحقيقه الشركات المصنعة للسيارات اذ يؤثر بصورة مباشرة في معدل المبيعات لذلك كان من الواجب تتبع العوامل التي تؤثر على درجة الراحة لمستخدمي المركبة ، وفي هذا المجال تم التركيز على السائق دون باقي شاغلي المركبة لانه المسؤول عن قيادتها وسلامة الركاب وكذلك لكي يتمكن من التركيز على الطريق وملاحظة العقبات امامه ويتم تحقيق ذلك بعزل بدن المركبة عن الاهتزازات الناتجة من عدم انتظام الطريق بواسطة منظومة تعليق مناسبة لحمل المركبة ونوع الطريق الذي تسير فيه ولأجل الحصول على نتائج دقيقة تم دراسة التصرف اللا خطي لأجزاء المنظومة (Non – linear behavior) أخذين بنظر الاعتبار إحدى أبرز مصادر التصرف اللا خطي لمنظومة التعليق المتمثلة باعتبار زاوية التآرجح (Pitch) كبيرة والتصرف اللا خطي لكل من النايبض والمخمد . ولهذا الغرض تم اختيار نموذج مناسب وذلك بأخذ نموذج نصف مركبة حمل (Truck Vehicle) لا خطي لأربع درجات من الحرية (4 DOF) وباعتبار جسم المركبة صلباً (Rigid Body) تم إثارة النموذج باستخدام نوعين من العقبات نصف الموجة الجيبية (half Sin Wave) والشبه منحرف (Trapezoidal) كتعبير عن تعرجات الطريق الذي تتعرض له المركبة أثناء السير وتم انشاء برنامج حاسوبي بلغة فورتران لغرض التشبيه (Simulation) وبعد الحصول على النتائج المتمثلة بالإزاحة والسرعة والتعجيل في مركز كتلة البدن (Center of Gravity) تم إجراء مقارنة مع نتائج بحوث نشرت حديثاً وتبين مطابقتها بدرجة مقبولة ولغرض معرفة مدى التحسن الحاصل عند الأخذ بنظر الاعتبار للعوامل التي تؤدي الى التصرف اللا خطي للمنظومة قمنا بالمقارنة مع النموذج الخطي لنصف مركبة حمل وتبين ان اعتماد اللا خطية يؤدي الى تحسين النتائج. بعدها نظرنا الى موضوع راحة السائق لكي نعلم ما تحقق لدينا من نتائج في هذا المجال اذ تم اختيار معيار مستوى راحة الراكب (Ride Comfort Level) ووظفت النتائج التي توصلنا اليها لتحديد السرعة القصوى التي تبقى المركبة ضمن حدود الراحة باستخدام ثلاثة انواع من العقبات . حيث تبين ان اعتماد اللا خطية في الحسابات يعطينا مدى سرعة أكبر . وبهذا نستطيع القول اننا حققنا الهدف من هذا البحث.

**Nomenclature**

$A_{rms}$	root mean square acceleration, (m/s <sup>2</sup> )
$A_{ref}$	reference root mean square, (m/s <sup>2</sup> )
$c_{sf}$	coefficient of damping of front damper, (N.s/m)
$c_{sr}$	coefficient of damping of rear damper, (N.s/m)
$c_{uf}$	coefficient of damping of front tire, (N.s/m)
$c_{ur}$	coefficient of damping of rear tire, (N.s/m)
$f$	force influential on the spring, (N)
$I$	inertia of suspension system, (kg.m <sup>2</sup> )
$F$	total load on the vehicle, (N)
$K$	spring stiffness, (N/m)
$K_b$	Stiffness of bump stop, (N/m)
$K_t$	stiffness of tire, (N/m)
$k_{sf}$	stiffness of front spring, (N/m)
$k_{sr}$	stiffness of rear spring, (N/m)
$k_{uf}$	stiffness of front tire, (N/m)
$k_{ur}$	stiffness of rear tire, (N/m)
$L_1$	distance of front tire from C.G of vehicle, (m)
$L_2$	distance of rear tire from C.G of vehicle, (m)
$M_s$	mass of suspension parts, (kg)
$m_{uf}$	mass of front tires group, (kg)
$m_{ur}$	mass of rear tire group, (kg)
$M_z$	moments around (z) axis, (N.m)
$P_t^{(n)}$	stroke of bump stop, (m)
$X$	displacement in (x) direction, (m)
$Y$	displacement in y direction, (m)

**Greek symbols**

$\Delta x$	vertical springs displacement, (m)
$\Delta \dot{x}$	vertical velocity of damper, (m/s)
$\Delta S_\theta$	angular displacement in ( $\theta$ ) direction, (rad)
$\delta$	displacement, (m)
$\dot{\delta}$	velocity, (m/s)
$\ddot{\delta}$	Acceleration, (m/s <sup>2</sup> )
$\theta$	angle of rotation about (Z) axis, (deg)
$\theta_0$	initial condition of ( $\theta$ ), (deg)
$\alpha$	coefficient of non - linear damper
$\beta$	coefficient of non – linear spring

**Subscripts**

[ ]	Matrix
{ }	Vector

**1. INTRODUCTION**

After the development of the design of transport vehicles, the ride comfort has become a priority to be achieved [1], especially since the driver spends a long time driving this type of vehicle because of the long distances has to traveled [2] so it was necessary to provide the requirements of rest during driving in the way that contains multiple forms of obstacles that lead to excite the vehicle and generate unwanted vibrations that effect on driver comfort and reduce the life of the suspension system [3], then the suspension system must absorb this vibrations by resist it by springs and damped it by dampers [4]. In order to achieve this goal, we must increase the flexibility of the springs but this is not suitable for this kinds of cars because it leads to increase elastic deformation of the suspension system which leads to reduce the load that can be transported, and the increasing of stiffness of springs leads to reduce the system response to the small obstacles, so

balanced must be achieved between these two demands [1]. In the transport vehicle the leaf springs are used which made of steel sheets have wide loading range because of its high stiffness which increase the adhesion of the vehicle to the ground and prevent slipping. This kind of springs doing the task of damping through the movement of sheets on each other because the friction. The vibration absorption process is carried out by dampers, which are considered to be the important parts of suspension system, they reduce the speed of the vertical displacement between the chasses and rides compartment [5]. When the vehicle vibrates because of irregularity of the road the dampers begin to work which consists hydraulic resistance to the movements of the fluid which will dispersing some of the resulting kinetic energy and thus damping the vibration and absorbing it [4]. In depending of the effect of the damping the dampers are divided on to two types, the first work in one direction (i.e., work during the expansion stroke only) and the second type works in two directions (i.e., works during expansion and compression strokes), The damping ratio depends on the interval resistance of the fluid movement inside the damper due to friction, the indicator of the damping elements are good if it reduce the vibration rang by three to five times per cycle (up and down) [6]. In 2002, Etman [1] and others studied the non- linear behavior of the suspension system transport vehicle and took the non-linear behavior of the damper only, the object of the study is to design an additional damper placed in the central area of the front axles to absorb vibration resulting from the high rigidity of the system. In 2012, Hussain [3] and others also studied the suspension system but they took the non – linear resulting from the spring only, the focus of the study was to find the optimal damper that achieves the best comfort in vehicle, and show that by increasing the coefficient of damping the ride comfort will increase but to a certain extent, after this degree the damper will acting as a rigid body that transmits the vibration directly from the frame to the body of the vehicle which adversely effects on the ride comfort [3].

**2. THEORY**

To represent any moving system, the coordinates should be determined first as shown in Fig. 1. Then the mathematical model must be configured to represent the movement of all the blocks of the system.

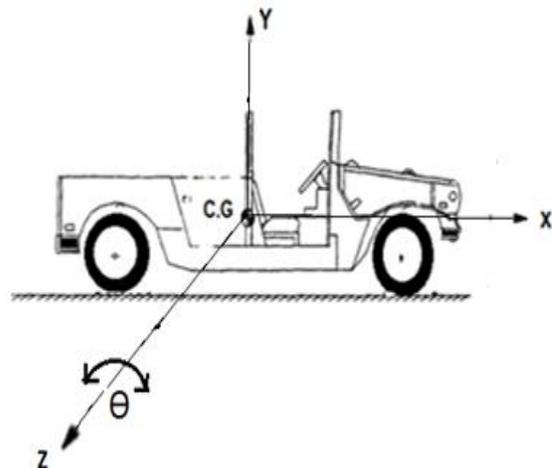


Fig. 1. The coordinate of vehicle [6].

In this model the reciprocating and fluctuating movement of the suspended blocks (chaises of vehicle) and the reciprocating movement of unsuspended blocks (tires) as shown in Fig. 2. To represent the non-linear behavior of the suspension system mathematically we should study the factors that lead to the linearity.

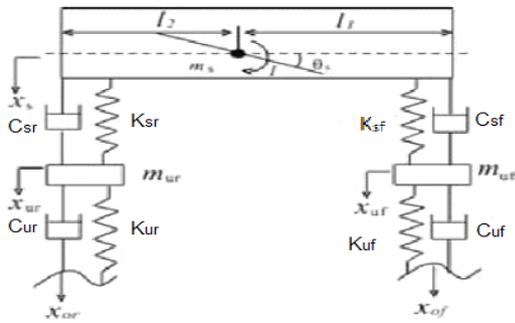


Fig. 2. Mathematical model of (2D) halve vehicle.

$$(F_y)_0 = \sum_{n=0}^n [c^{(n)}(\delta'_{0'} - \delta'^{(n)}) + k^{(n)}(\delta_0^{(n)} - \delta^{(n)})] + M_s \delta''_0 \tag{2}$$

Then

$$F_y^{(n)} = c^{(n)} [\delta'_{0'} - \delta'^{(n)}] + k^{(n)} [\delta_0^{(n)} - \delta^{(n)}] \tag{3}$$

After compensation magnitude of \$(\delta\_0, \delta'\_{0'})\$ in Eq. (1) we conclude that the forces in (Y) direction is: Because (n) in the chaises takes the symbol (0) then:

$$M_z = I_{zz} \theta''_0 + \sum_{n=1}^n [c^{(n)} (\delta'_{0'} - \delta'^{(n)}) (x_n - x_0) + k^{(n)} (\delta_0^{(n)} - \delta^{(n)}) (x_n - x_0)] \tag{7}$$

when

$$M_x^{(0)} = I_{zz} \theta''_0 \tag{8}$$

$$M_x^{(n)} = c^{(n)} (x - x_0) [\delta'_{0'} + \{(x - x_0) \cos \theta_0 - (y - y_0) \sin \theta_0\} \theta'_{0'} - \delta'^{(n)}] + k^{(n)} (x_n - x_0) [\delta_0^{(n)} + (x - x_0) \sin \theta_0 - (y - y_0) (1 - \cos \theta_0) - \delta^{(n)}] \tag{9}$$

**Third Equation**

It represents the equation of motion of tires in the y-direction. The force influential on tires can be calculated as follow [7]:

$$F_y = M^{(n)} \delta''^{(n)} + c^{(n)} [\delta'^{(n)} - \delta'_{0'}] + k^{(n)} [\delta^{(n)} - \delta_0^{(n)}] + c_T^{(n)} [\delta'^{(n)} - \delta'_{T'}] + k_T^{(n)} [\delta^{(n)} - \delta_{T'}] \tag{11}$$

By substituting Eqs. (A-1) and (A-2) from appendix in Eq. (4) and rearrange the resulting equation we get:

$$F_y = M^{(n)} \delta''^{(n)} + [c^{(n)} + c_T^{(n)}] - c^{(n)} [\delta'_{0'} + \{(x - x_0) \cos \theta_0 - (y - y_0) \sin \theta_0\} \theta'_{0'}] - c_T^{(n)} \delta'_{T'} + [k^{(n)} + k_T^{(n)}] \delta^{(n)} - k^{(n)} [\delta_0^{(n)} + (x - x_0) \sin \theta_0 - (y - y_0) (1 - \cos \theta_0) - \delta^{(n)}] - k_T^{(n)} \delta_{T'} \tag{12}$$

**Fourth Equation**

$$(R_y)_n = c_T^{(n)} [\delta'_{T'} - \delta'^{(n)}] + k_T^{(n)} [\delta_{T'} - \delta^{(n)}] \tag{13}$$

Thus the equation of motion of chaises and the suspension system can be represented as:

**2.1. Relative Increasing in the Swing Angle**

Fluctuating movement when imposes that the linear disposition to be a small swing angle \$(\theta)\$. The influence in results is small but when it is considered non – linear the angle of swing cannot have considered to be small and this will result in influence in the governing equations of motion as in the Fig. A-1 in the appendix, the swing angle is shown as a result of movement around (Z) coordinates as shown in appendix (A). Using Newton second low the equation of motion was derived as follows:

**First Equation**

This equation represents the movement of the body in the (y-axis) direction, the summation of forces exerted in the direction on the body in the (Y) direction is [7]:

$$(F_y)_0 = \sum (F_y)_{body} \tag{1}$$

$$F_y^{(0)} = m_s \delta''_0 \tag{5}$$

**Second Equation**

Represent the angular movement about z- axis. The summation of momentum acting about z-axis is [7]:

$$M_z = \sum (M_z)_{body} \tag{6}$$

This equation is based on the balance of forces at the point of contact between the tire and street: After substituting magnitude of both velocity and displacement we get:

$$F_y = \sum (F_y) \tag{10}$$

$$\begin{Bmatrix} (F_y)_0 \\ M_z \\ (R_y)_1 \\ (R_y)_2 \end{Bmatrix} = [M] \{ \delta'' \} + [C] \{ \delta' \} + [K] \{ \delta \} \tag{14}$$

when

$$\delta = \{ \delta_0, \theta_0, \delta^{(1)}, \delta_T^{(1)}, \delta^{(2)}, \delta_T^{(2)}, \dots \dots \}$$

Let  $\Delta x_n = (x_0 - x_n)$ ,  $\Delta y_n = (y - y_0)$

$$K^n = \begin{bmatrix} k & k[\Delta x_n \sin \theta_0 - \Delta y_n (1 - \cos \theta_0)] & -k & 0 \\ \Delta x_n k & k[(\Delta x_n)^2 \sin \theta_0 - \Delta y_n (\Delta x_n)(1 - \cos \theta_0)] & -\Delta x_n k & 0 \\ -k & -k[\Delta x_n \sin \theta_0 - \Delta y_n (1 - \cos \theta_0)] & k + k_T & -k_T \\ 0 & 0 & -k_T & k_T \end{bmatrix} \tag{15}$$

$$C^n = \begin{bmatrix} C & C[\Delta x_n \cos \theta_0 - \Delta y_n \sin \theta_0] & -C & 0 \\ \Delta x_n C & C[(\Delta x_n)^2 \cos \theta_0 - \Delta y_n \Delta x_n \sin \theta_0] & -\Delta x_n C & 0 \\ -C & -C[\Delta x_n \cos \theta_0 - \Delta y_n \sin \theta_0] & c + c_T & -c_T \\ 0 & 0 & -c_T & c_T \end{bmatrix} \tag{16}$$

The equation of equilibrium of the suspension system is:

$$[M]\{\ddot{\delta}\} + [C]\{\dot{\delta}\} + [K]\{\delta\} = \tag{17}$$

### 2.2. Non-Linear Behavior of Spring and Damper

Its known that the spring and damper are the base of any suspension system so any non –linear behavior of one or both will effect on the performance of the suspension system.

#### Non-Linear Behavior of Spring

It is usually to deal with the spring as mechanical component that stored energy with constant stiffness that mean that the relationship between the force and deformation is a linear relationship as shown in Fig. 3(a) and we can use the equation:

$$F = K . \Delta x \tag{18}$$

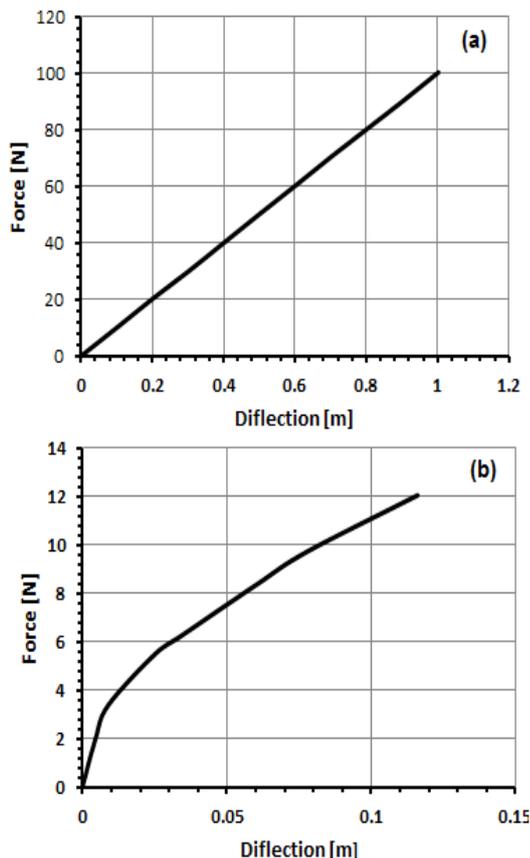


Fig. 3. The relationship between force and displacement (a) linear, and (b) non-linear.

Then the matrix of coefficient of spring and damper will b

But the real behavior is quite deferent since the relationship between the force and deformation is non-linear as in Fig. 3(b), that mean there is no fixed value of stiffness (K) which change with the change of exerted load, so that the force is a function of deformation [7] as in the Eq. (19).

$$F = f (\Delta x) \tag{19}$$

After conducting experiments by the researchers Hussain [3] and others they reach to the equation which will be used in

$$F = K \Delta x + \beta K \Delta x^2 \tag{20}$$

when  $\beta = 0.3$  [3].

#### Non-Linear Behavior of Damper

The damper is one of the part that behave away from linearity but this is not similar to the direction of the piston movement, this non – linearity acts a force in rebound stroke greater than the compression stroke [8] as shown in Fig. 4(b).

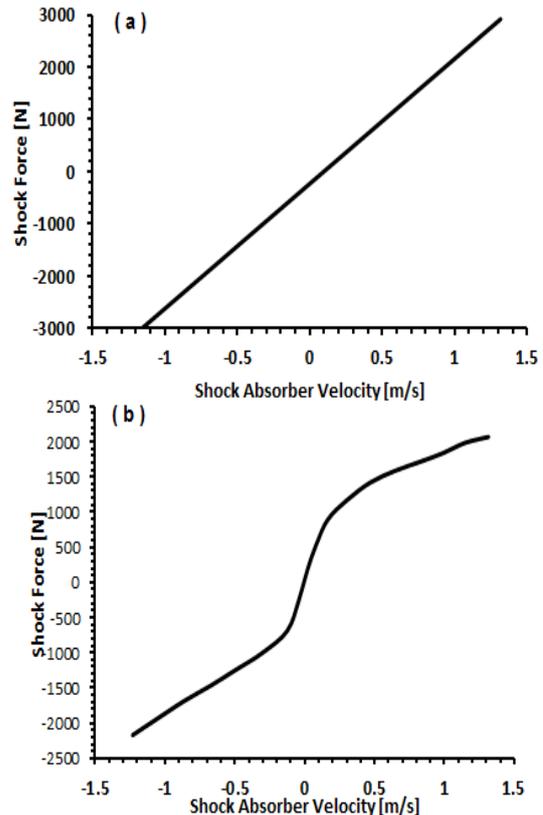


Fig. 4. The relationship between force and velocity (a) linear, and (b) non-linear.

In general, the damper shows this behavior in two cases: When the wheel shocking an obstacle suddenly then the damper can't absorb the resulting shock completely then the sum of shock will transmit to the upper section of vehicle which adversely effects on the ride comfort [9].

When the vehicle is moving on the road which contain little roughness, which leads to non – response of the damper and will behave as rigid body, this behavior is because two reasons:

The first is high stiffness of the spring which controls the valve movement of the damper then the pressure of the oil resulting from the vertical movement of the piston is not sufficient to open the valves, this leads to limited response of the vehicle to small obstacles.

The second is friction between the piston and the cylinder then the piston dose not move unless the force resulting overcome the friction force [10]. The equation that use to express the non-linear behavior of the damper is:

$$f_y = m^{(n)}\delta'' + c^{(n)}[\delta'^{(n)} - \delta_0'^{(n)}] + (k^{(n)} + k_b^{(n)})[\delta^{(n)} - \delta_0^{(n)}] + c_T^{(n)}[\delta'^{(n)} - \delta_T'^{(n)}] + k_T^{(n)}[\delta^{(n)} - \delta_T^{(n)}] + k_b[p_T^{(n)} - \delta_T^{(n)}] \tag{22}$$

**Comfort Criteria**

After the mathematically representation of non – linear behavior of the system and inter the resulting equations in the program used to simulate their results (displacement, velocity and acceleration) in the center of mass of the body then it used to determine the degree of comfortable of passengers, to achieve this goal we adopted ride comfort level that depends on root mean square of acceleration and can be found by the Eq. [3].

$$RCL = 20 \log_{10} \frac{A_{rms}}{A_{ref}} \tag{23}$$

Korea this criterion was adopted to evaluate the vehicles performance by creating table that links it with comfort level

**Table 1**  
relationship between RCL and comfort degree [2].

RCL (db)	Ride Comfort
103	Very comfort
103 – 108	comfortable
108 – 113	Medium
113 – 118	uncomfortable
118	Very uncomfortable

Depending on this table can be considered to be (113 db) is the maximum value acceptable to achieve driver comfort.

**3. RESULTS AND DISCUSSION**

When a mathematical model is derived and computer program is built for the purpose of simulation, a comparison should be made with newly published research to determine the accuracy and their reliability. By compare with halve car model used from Ref. [1] using two kinds of obstacles, the first is trapezoidal obstacle its height (0.25 m) and width (2.25 m), from the results explained in Fig. 5(a) we note that the maximum acceleration of the vehicle body is (14.7 m/s<sup>2</sup>) and in Fig. 5(b) which represent

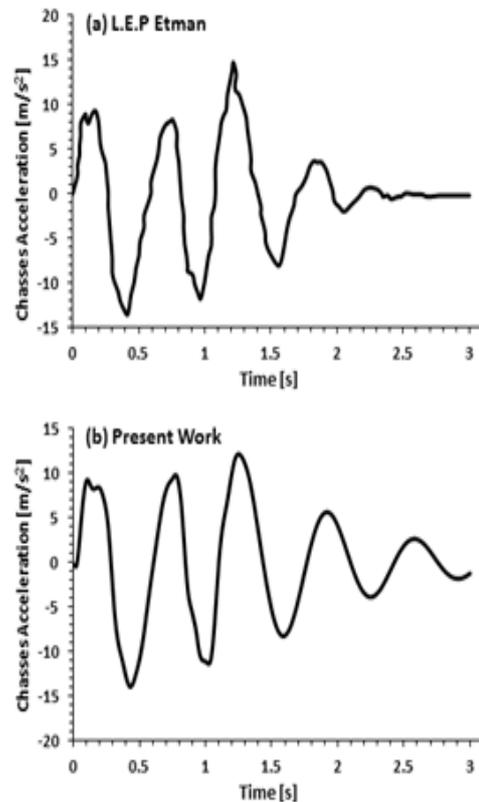
$$F = C.\Delta x + \alpha C(\Delta x)^2 \tag{21}$$

where  $\alpha = 0.38$  [1].

**Bump Stop**

As a result of the irregularity of the road, one or more of the tires will leave the ground towards the body of the vehicle then we need for a bumper to identify this additional movement and return the tire to its position in contact with the street to prevent slippage and prevent collision the tire with the bottom of the body of the vehicle this bumper is called (bump stop), it makes usually of rubber and installed at the top part of the chases. When the bump stop shock the bottom of the body an additional force will generated which added to the resultant forces that effect on the car body and cause disturbance [9] then the Eq. (22) will be:

the results obtaining using the program prepared for this purpose the maximum acceleration is (12.3 m/s<sup>2</sup>) which decreased by (16%).



**Fig. 5.** Relationship of vertical acceleration and time at speed 15 km/hr.

The second obstacle used is halve sin wave its amplitude (0.5 m) and wave length (6 m), from the results that explained in Fig. 6(a) the maximum vertical acceleration of the vehicle body is (13.4 m/s<sup>2</sup>) but in Fig. 6(b) it drops to (12.6 m/s<sup>2</sup>) this indicates the improvement of the results when adopt an additional source of non-linear behavior of the suspension system.

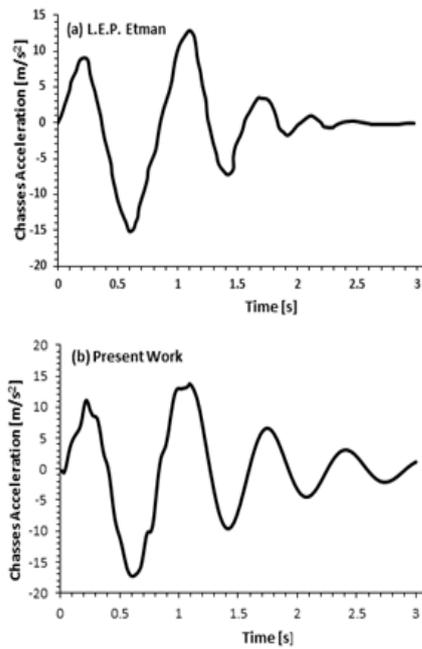


Fig. 6. Relationship of vertical acceleration and time at speed (80 km/hr).

Table 2

Data of halve truck vehicle [3].

$M_s$ (kg)	$I$ (k g.m <sup>2</sup> )	$M_{uf}$ (kg)	$M_{ur}$ (kg)	$C_s$ (N.s/m)	$C_u$ (N.s/m)	$K_{sf}$ (N/m)	$K_{sr}$ (N)	$K_{uf}$ (N/m)	$K_{ur}$ (N/m)	$L_1$ (m)	$L_2$ (m)
7000	30000	600	900	500	1000	450000	700000	3000000	3000000	3.2	-1.8

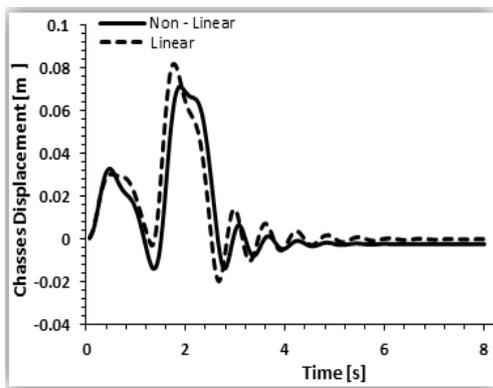


Fig. 7. Relationship of vertical displacement and time at speed (30 km/hr).

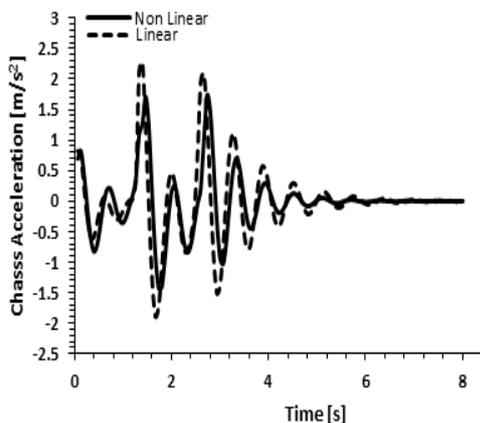


Fig. 8. Relationship of vertical acceleration with time.

From the last Figs. (5) and (6) found that the results obtained by using the prepared program approach by acceptable degree to the researcher results adopted to compare. To find out the change in results when adoption of the non-linear factors a comparison with the linear results have been doing with the linear model used from Ref. [4] by adoption the data in Table 2.

First comparison is with the vertical displacement of body which explained in Fig. 7 using halve sin wave obstacle it's amplitude is (0.2m) and wave length (1m) it's found that when using non-linear model the maximum displacement of the body is (0.066m) but when using linear model is (0.082) this indicates that a decrease in vertical displacement (19.5%) when adopting non-linearity, which increase the degree of ride comfort.

The studies showed that the vertical displacement can't give an impression of the ride comfort alone [10] therefore we compare by using vertical velocity and acceleration of body. The results in Fig. 8 shown that by non-linear model the vertical acceleration of the body is (1.7 m/s<sup>2</sup>) and by using linear model the maximum acceleration is (2.3m/s<sup>2</sup>) that mean (26%) reduction in acceleration when adopting non-linear model which also increase the ride comfort.

this improvement in the results when considering the factors that leads to the non – linear behavior of the suspension system, we used the results to know what achieved in the ride comfort when adopt the non – linear factors by using the limited speed of vehicle which remains the vehicle within the comfort limit by using three tips of obstacle and by use ride comfort level criteria in Table 1 and data of truck vehicle in Table 2. From the results shown in Fig. 9 when using the first obstacle (trapezoidal type) (0.1m) height and( 0.3) width , the vehicle remains within the comfort limit to speed (84 km/hr) , but when using a halve sine wave it's amplitude (0.1) and wave length (1 m) the speed limit increase to (93 km/hr) and with decreasing

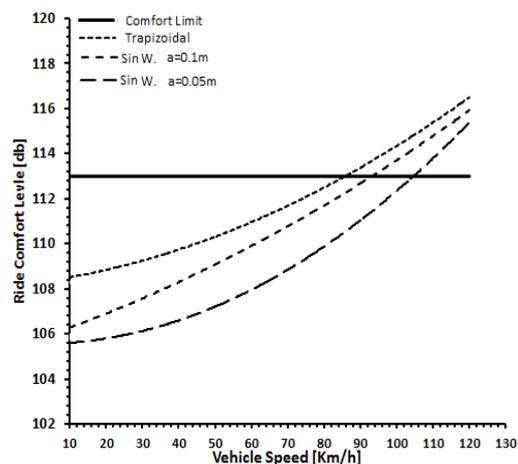


Fig. 9. relationship of vehicle speed with ride comfort level using different obstacles.

the second amplitude of the obstacle to (0.05 m) the speed limit increase to (107 km/hr) that leads to the last half sine wave give larger range than the other two obstacles, this is logical result. For the purpose of using ride comfort level criteria to determine the improvement in results when adopting nonlinearity, we compared the linear and non-linear models of the same previous relationship using half sine wave obstacle amplitude (0.1 m) and wave length (1 m) as shown in Fig. 10, it turns out that the use of non-linear model gives us velocity range larger than the linear model until (100 km/hr) but in the linear model the velocity range until (75 km/hr) .

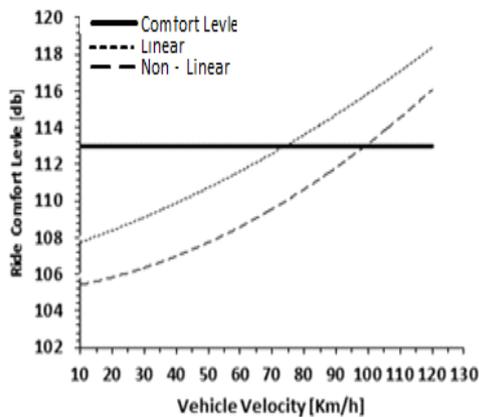


Fig. 10. Relationship of velocity and ride comfort level.

#### 4. CONCLUSIONS

After ascertaining the accuracy of the results by comparing with newly published researches and finding out the improvement that occurred when adoption of the non – linear behavior of the suspension system we found the following:

1. Two dimensional half vehicle can be used to represent vehicle behavior when exposed to road irregularity because of its easy and accuracy in the results.
2. The selection of the non – linear factors was successful because of change in the results towards improvement which achieve ride comfort.
3. The speed of the vehicle can be used as a function of comfort in Known working conditions and external excitation by adopting ride comfort level criteria.
4. The computer program prepared for the results simulators can be used to understand the performance of the suspension system which saving money and effort in building the zero model.

#### 5. RECOMMENDATIONS

It is possible to develop this study by following one or more of the following recommendations:

1. Conduct a study on the possibility of completely isolating the drivers compartment from the body of the

vehicle using a separate suspension system characterized by high flexibility.

2. Conducting a study on the possibility of using active or semi-active suspension system in the truck vehicle.
3. Dependence of random excitation in study of the system.
4. Adopt full car model 3D.
5. Conduct a practical study and compare it with the theoretical results to make sure of the results.

#### REFERENCES

- [1] Etman LFP, Vermeulen RCN, Van Heck JGAM, Schoofs AJG, Van Campen DH Vermilion. Design of stroke dependent damper for the front axle suspension of truck using multibody system dynamics and numerical optimization. *Vehicle System Dynamics* 2002; **38** (2) 85-101.
- [2] Brogioli M, Gobbi M, Mastinu G, Pennati M. Parameter sensitivity analysis of passenger seat model for ride comfort. *Experimental Mechanism* 2011; **51** (8): 1239 -1247
- [3] Zahib H, Nurul-Absar K. Ride comfort of 4-degree of freedom non-linear heavy vehicle suspension. *ISESCO Journal of Science and Technology* 2012; **8** (13): 80-85.
- [4] Saif M. A tow dimensional vehicle using to study suspension system and its influence into driver comfort. MSc. Thesis, Tikrit University; Iraq: 2013
- [5] Muslim T. Suspension system in vehicle, volume 1, Aplide scinde, Arabic Encyclopedia, 2012.
- [6] Leonard M. Fundamental of vibration. Virginia Polytechnic Institute and state university, USA; 2001.
- [7] Yan C, Kurfess TR, Messman M. Testing and modeling of non-linear properties of shock absorbers for vehicle dynamic studies. *San Francisco* 2012; **II**: 978- 988.
- [8] Salim Y. Ride analysis for suspension system of off-road tracked vehicle. Ph.D. Thesis, Cornfield Institute of Technology Gen; UK: 1991.
- [9] Ard Bomer. Non-linear model for comfort analysis. MSc. Thesis, Dynamics and Control Group, Technique University Eindhoven; 2006.
- [10] Caven I, Kay N, Oztruk F. Design tool to evaluate the vehicle ride comfort characteristics: Modeling, physical testing and analysis. *The International Journal of Advance Manufacturing Technology*, 2012; **60**: 755-763.

#### APPENDIX

##### Effect of Rotation about Z-Axis

Consider the rotation about (z) axis by angle ( $\theta$ ), considering a point in space ( $a$ ) and because of rotation about z-axis it will be ( $a'$ ), then we can describe the body movement in y-direction as:

$$\Delta s_{\theta} = r \sin(\theta_0 + \alpha) - r \sin \alpha = r(\sin \theta_0 \cos \alpha + \cos \theta_0 \sin \alpha) - r \sin \alpha \quad (A - 1)$$

On the assumption that:

$$\left. \begin{aligned} r \sin \alpha &= (y - y_0) \\ r \cos \alpha &= (x - x_0) \end{aligned} \right\} \quad (A - 2)$$

we can conclude that  $\Delta s_{\theta}$  equal:

$$\Delta s_{\theta} = (x - x_0) \sin \theta_0 + (y - y_0) \cos \theta_0 - (y - y_0) = (x - x_0) \sin \theta_0 - (y - y_0) (1 - \cos \theta_0) \tag{A - 3}$$

**Effect of C.G Movement Towards the Movement  $\delta_0$  and Swing  $\theta_0$**

$$\sin \theta_0 - (y - y_0) (1 - \cos \theta_0) \tag{A - 4}$$

The movement in (y) direction in the center of body (C.G) as in Fig. 3 we can express it as:

$$x - x_0 + \delta(x, y, t) = \delta_0$$

Equation (1) describe the vertical displacement ( $\delta_0$ ) because of the swing by deriving Eq. (1) we get:

$$\delta'(x, y, t) = \delta'_0 + (x - x_0) \cos \theta_0 \theta'_0 - (y - y_0) \sin \theta_0 \theta'_0 \tag{A - 5}$$

$$\delta'(x, y, t) = \delta'_0 + \left[ \frac{(x - x_0) \cos \theta_0 - (y - y_0) \sin \theta_0}{\theta'_0} \right] \tag{A - 6}$$

By the same way but deriving velocity ( $\delta'$ ) in Eq. (2) we get the acceleration ( $\delta''$ ):

$$\delta''(x, y, t) = \delta''_0 + [(x - x_0) \cos \theta - (y - y_0) \sin \theta] \theta''_0 - [(x - x_0) \sin \theta + (y - y_0) \cos \theta] \theta'_0 \tag{A - 7}$$

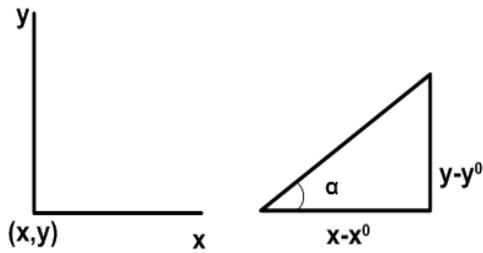
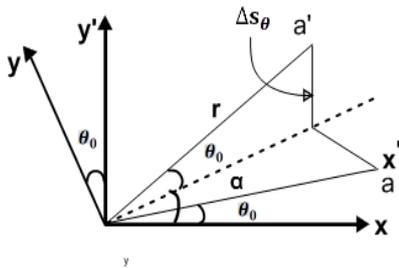


Fig. A.1. Rotating about z-axis.