## ESTIMATION OF MINIMUM COST DESIGN OF CIRCULAR GRAIN SILO

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## ABSTRACT

This study is an application of optimization method to the structural design of circular silo, considering the total cost of the silo as an objective function of the properties of the silo and stored materials (unit weight of stored materials, angle of inclination of hopper wall, height of silo, height of hopper and silo diameter), as a design variables.

A computer program has been written to solve numerical examples using the ACI code equations and all new requirements and criteria in concrete design.

It has been proved that the minimum total cost of the silo increases with the increase of the angle of internal friction between stored materials, the coefficient of friction between stored materials and concrete, and the number of columns supporting hopper.

#### **KEYWORDS**

Optimization, structural design, ACI code

## NOTATIONS

- Ar area of cross section of ring beam
- A<sub>s</sub> reinforcement steel area (per unit width of wall)
- C<sub>d</sub> overpressure factor, for static pressures to design pressures
- D inside diameter (unless noted)
- E<sub>s</sub>,E<sub>c</sub> modulus of elasticity for steel and concrete ,respectively
- $F_m$  meridian membrane force per unit width of hopper wall
- Ft horizontal membrane force per unit width of hopper wall
- f<sub>y</sub> yield strength of steel
- H height of silo
- H<sub>A</sub> shear in ring beams at face of columns
- I moment of inertia
- L perimeter of inside cross section
- M bending moment, per unit width of wall
- P horizontal static pressure due to stored material
- R hydraulic radius of horizontal cross section of storage space
- V<sub>u</sub> ultimate shear force
- W total weight of stored material
- Y depth from surface of stored material to design point
- des a subscript indicating 'design ' force or pressure
- $f_{c,vert}$  vertical compressive stress at wall bottom
- $f_c^{'}$  ultimate compressive strength of concrete
- $f_s$  steel stress ,tension
- h<sub>c</sub> height of column
- L.L. a subscript meaning 'live load'

- m shrinkage coefficient, generally 0.0003
- n modular ratio ,Es/Ec
- q static vertical pressure due to stored material
- $q_{\alpha}$  unit static pressure normal to a surface inclined at angle  $\alpha$
- $\alpha$  angle of inclination of hopper wall
- $\gamma$  unit weight of stored material
- μ coefficient of friction between stored material and wall
- $\mu$  =tan  $\phi$
- $\phi$  angle of internal friction
- $\Psi$  capacity reduction factor

#### **INTRODUCTION**

Bins for storing granular materials are of two main types silos (also called deep bins), and bunkers (or shallow bins). The important difference between the two is in the behavior of the stored materials. This behavior difference is influenced by both bin geometry and characteristics of the stored material. Material pressures against the walls at bottom are usually determined by one method for silos and by another for bunkers.

Silos and bunkers are made from many different structural materials. Of these, concrete is probably the most frequently used. Concrete can offer the necessary protection to the stored materials, requires little maintenance, is aesthetically pleasing, and is relatively free of certain structural hazards, which may be present in silos or bunkers of thinner materials. Silos and bunkers may be of various plan shapes and may occur singly or connected in-groups. See Figures  $(1,2)^{[1]}$ .

#### **PURPOSE OF STUDY**

The purpose of this study is to detect the capabilities of optimization method to handle the economical structural design of a circular grain silo, giving a safe design based on considering the effects of different parameters on the silo cost and giving the designer the relationships and curves between variables of design so that the design of a circular grain silo can be more reliable and simple.

#### HISTORICAL BACKGROUND

Torres et al., (1966) presented the minimum cost design of prestressed concrete highway bridges subjected to AASHTO loading by using piecewise LP method <sup>[2]</sup>.

Kirsch (1972) presented minimum cost of a continuous twospan prestressed concrete beam .The cost function included only cost of concrete and cost of prestressing steel <sup>[3]</sup>.

Namman (1982) presented minimum cost design of prestressed concrete tensile member based on the ACI-Code 1977 .The cost function includes the material costs of concrete and the prestressing steel <sup>[4]</sup>.

Al-Jubair (1994) minimized the cost of ring foundations by using simplex method of Nelder and Mead. The results obtained supported the efficiency of optimization techniques in selecting the most economical design of ring foundations for given conditions<sup>[5]</sup>.

Al-Douri (1999) minimized the cost of rectangular combined footings by using several methods .She concluded that the minimum cost of the footing decreases with increasing the distance between the columns for a constant length <sup>[6]</sup>.

Al-Jubori (2001) minimized the cost design of mat foundations .He proved that the minimum cost of the raft foundation decreases with increasing of the angle of internal friction of soil and increases with increasing the column spacing in both directions as well as with increasing the difference between the loads of adjacent columns<sup>[2]</sup>.

#### **OBJECTIVE FUNCTION**

The total cost of silo can be represented by:

#### ZT=CSRE+CSFW+CSCO .....(1)

Where:

ZT= total cost (unit price).

CSRE= cost of silo reinforcement (unit price).

CSFW= cost of silo formwork (unit price).

CSCO= cost of silo concrete (unit price).

CSRE= TOTRE\*COR

={ $\pi$ DH [(1/SPS<sub>av</sub>)+(1/SPS<sub>sv</sub>)]+A<sub>sh</sub> }\*COR .....(2)

CSFW= Awohle\*COFW  
= 
$$\{\pi D H * 2 + \pi D (Hh \sin \alpha)\}$$
\*COFW .....(3)  
CSCO= V<sub>wohle</sub>\*COCO

= 
$$\{A_{wohle} * t + V_{co}\} * COCO$$
 .....(4)

Where:

TOTRE= Total weight of reinforcement steel (Ton)

COR= Price of reinforcement (unit price/Ton)

COFW = Price of formwork (unit price/m<sup>2</sup>)

COCO= Price of concrete (unit price/ $m^3$ )

D, H, Hh, t = Silo diameter, height, hopper height, and thickness of wall, respectively.

 $SPS_{av}$ ,  $SPS_{sv} = Average$  Spacing between horizontal steel,

and that between vertical steel, respectively.

 $A_{sh}$  = Amount of hopper reinforcement.

 $V_{co}$ = Volume of columns concrete.

#### **PROPERTIES OF STORED MATERIALS**

The properties of material to be stored affect the intensity of loading pressure, in addition, they influence on material flow and must be considered in selecting the outlet shape and size and the type of unloading system. ACI 313R-97, Table  $4-A^{[7]}$ , suggests stored material properties. Here in shown in table (1).

#### STATIC PRESSURES-LATERAL AND VERTICAL

The initial vertical pressures at depth y below the surface of the stored material is given as (see Figure 3):

Where; R is the ratio of area to perimeter horizontally cross-section of storage space. For a circular silo R = r/2.

The initial horizontal pressure at depth y below the surface of the stored material can expressed as:

$$P = (1 - \sin \phi) q$$
 .....(6)

The design horizontal pressure on the wall above the hopper for concentric flow patterns is obtained by multiplying the initial pressure p by a minimum overpressure factor  $C_d$  of 1.5. The required strength force per linear meter of height of wall is given by equation (7) (see Figure 4):

Where L.L =1.6 is a factor of safety for live load (ACI 318-02)<sup>[7]</sup>, and  $\phi = 0.9$  is the strength reduction factor for axial tension as suggested by ACI 318-02.

The term static pressure applies only, for stored material at rest (i.e., before withdrawal is begin). During withdrawal, these pressures may increase. The increases are sometimes called dynamic effects, but the term "overpressure" is preferred since the increases include both static and dynamic effects. However, its effect can be approximated using overpressure factor  $C_d$ , to convert from computed static pressure to designed pressure. In general.

Design pressure = 
$$C_d$$
 \* static pressure ......(9)

For silos with centrally located discharge opening, design pressures due to stored material are:

#### **Conical Hopper**

The design pressure,  $q_{\alpha,des}$  may be computed from Eq (12). The conical hopper shell is subjected to two tensile membrane forces. The meridian force,  $F_m$ , is parallel to the generator line of the cone. The tangential force,  $F_t$ , is in the plane of the shell and horizontal. These forces, shown in Fig. (5), are the resultant of vertical pressures,  $q_{des}$  (at depth Y) and W, the combined weights of the hopper itself and material stored below depth Y plus any equipment supported by the hopper.

The required reinforcement area per unit width of shell is:

A conical hopper is usually supported at its upper end by a ring beam. The depth of the ring beam should not be less than one-tenth of hopper diameter. See Fig. (6).

#### Wall Thickness

An isolated circular silo under uniform radial load gets its strength from the horizontal steel wherever the concrete is cracked. One approach is the PCA formula, Eq. (17) below:

In this equation, PCA suggests using 0.1fc' for allowable stress  $f_{c',ten}$ . Vertical compressive stresses should also be checked. Suggested limits for circular silos are

 $f_{c,vert} = 0.385 \text{ fc}'$  where, fc' in MPa .....(18)

#### **Crack Control**

Wall thickness and reinforcement have to be so proportioned that, under pressure, the design crack width  $W_c$  computed using formula (19) shall not exceed 0.25 mm<sup>[7]</sup>:

$$W_c = 0.0001 f_s [dc A]^{1/3} (0.145) \leq 0.25 mm ...(19)$$

Where  $f_s$  in (MPa) represents the calculated stress in reinforcement steel at initial pressures, and dc and A are expressed as follows;

$$\left\{ \begin{array}{c} dc = 2.5 d_b \\ A = 2 dc \ s \end{array} \right\}$$
 .....(20)

Where  $d_b$  is the diameter of the horizontal steel bars.

#### **Ring Beam and Columns Supporting a Hopper**

The concrete ring beam must be designed to carry all loads including torsion moments. The external design loads acting on the ring beam (shown in Fig. 7-9) are approximately calculated as;

Where: gr is the self-weight of ring beam per unit length.

$$M_t = F_{mu} e$$
 (N.m/linear m) .....(23)

Coordinates of the centroid measured from origin O are:

$$\bar{x} = (a_1b_1^2/2 - (a_2b_2/2)(b_1-b_2/3))/Ar$$
 .....(24)

$$y = (a_1^2 b_1 / 2 - (a_2 b_2 / 2)(a_1 - a_2 / 3)) / Ar$$
 .....(25)

An equivalent rectangle (shown dotted in Fig. 9) of height 'a'and width 'b' is substituted for the pentagon:

$$a=2 y$$
  $b=A_r/a$  .....(26,27)

The column shear,  $H_A$ , and upper end moment,  $M_A$ , are found by solving simultaneously Eqs. (29) and (30):

$$F_{X}r^{2}/A_{r} = M_{A} \left[ h^{2}c/2I_{c} \right] - H_{A} \left[ h^{3}c/3I_{c} + \eta r^{3}/2I_{r} \right] \qquad \dots (28)$$

•

$$12M_{t} r/a^{3} \ln(r_{2}/r_{1}) = M_{A} [h_{c}/I_{c} + \pi r z/(6.8b^{4}\lambda)] -H_{A} [h^{2}_{c}/2I_{c}] .....(29)$$
  
Where  $\eta$  and z are numerical coefficients and  $\lambda$  is a torsional

property of the equivalent rectangular section <sup>[8]</sup>.

$$M_{B} = H_{A}h_{c} - M_{A} \tag{30}$$

The cross sectional area of ring beam is:  $A_r=a_1b_1-b_2a_2/2$  ....(31)

#### **COMPUTER PROGRAM**

The main program was utilized to perform the necessary calculations for optimization was drawn from Bundy (1984)<sup>[9]</sup> and translated to FORTRAN-77.Hooke and Jeeves method performed the minimization process utilizing this method of solution. Following are the required input parameters for this program.

Ns- number of independent (design) variables.

X(Iz)-initial estimate of the design variables [Iz=1,2,3,.....Ns] Hz-step length.

The program (Silo. For) in FORTRAN-77 is written by using the design procedure of ACI-Code with code improvement in load factors and crack width equation <sup>[7]</sup>. This program gave good results with code requirements and other design criteria.

The program (Silo. For) use a subroutine with the program (H & J. For). Input data symbols and other parameters used in

subroutine (Silo. For) is listed in table (2) and results shown in table (3).

## **Numerical Example**

The basic data of the problem is shown in Fig. (10) .The problem was solved using three initial trial values for design variables vector X=[ $\gamma$ , $\alpha$ , H, HH, D] .The input data is Ns=5 The first initial trial values: X(1)=9 , X(2)=45 , X(3)=35 , X(4)=7 , X(5)=11. The second initial trial values: X(1)=8 , X(2)=60 , X(3)=40 , X(4)=9 , X(5)=12 .The third initial trial values: X(1)=7 , X(2)=50 , X(3)=30 , X(4)=8 , X(5)=10 . Hz=0.01

The results obtained are shown in table (4). Figs (11-13) show the convergence rate towards the minimum cost design of circular grain silo.

#### **DISCUSSION OF RESULTS**

A parametric study was done to the angle of internal friction of the stored material, coefficient of friction between stored material and concrete wall and the number of columns supporting conical hopper for the first initial trial point. The results are listed in tables (5,6,7).

It can be observed from table (5) and Figs. (14-19) and Fig. (28) that as the angle of internal friction increases; the minimum total cost is increased, Fig (14). The increase is noticed after angle value of  $30^{\circ}$ , and minimum total cost is at  $25^{\circ}$ . The

optimum unit weight is slightly decrease after angle 30°, Fig. (15). The optimum hopper angle, silo height and hopper height are increased after the angle of 30°, Figs. (16,17,18). The maximum crack width remains constant and later slightly decreases after the angle was 30°, Fig. (19). The optimum silo diameter increases after the angle was 30°, Fig. (28). So, from the obtained results it is concluded that the optimum angle of internal friction is 25°.

It can be observed from table (6) and Figs (20-25) that as the coefficient of friction between stored material and concrete wall increases; the minimum total cost rapidly increased Fig. (20). The optimum hopper angle, silo height, silo diameter and hopper height also slightly increased, Figs. (22-25). But the optimum unit weight is little change, Fig. (21).

It can be realized from table (7) and Figs (26,27) that as the number of columns supporting conical hopper increase; the minimum total cost is approximately remain constant, but increased when the number of columns is twelve or more. The optimum hopper height and silo diameter slightly decreases when the number of columns increases. The other variables still constant when the number of columns increases.

## CONCLUSIONS

1-The economical structural silo design can be handled as a problem of mathematical programming.

- 2-Optimization techniques were powerful applied to the optimum structural silo design.
- 3-The minimum total cost was more sensitive to the changes in angle of internal friction between stored materials and coefficient of friction between stored materials and concrete wall.

4-Increase in angle of internal friction leads to increase minimum total cost , hopper height, silo height, hopper angle and silo diameter. This increase effects as Little decrease in unit weight of stored materials and maximum crack width. Optimum angle of internal friction is 25°.

- 5-Increase in coefficient of friction leads to increase minimum total cost, hopper angle, silo height, hopper height and silo diameter. So, little changes are obtained in unit weight of stored materials.
- 6-Increase in number of supporting columns of conical hopper leads to increase total cost and silo diameter.

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| Unit weight of stored material, $\gamma$ [kN/m <sup>3</sup> ] | 7-10      |
|---|-----------|
| Coefficient of friction between stored material               | 0.29-0.47 |
| and wall surface , $\boldsymbol{\mu}$                         |           |
| Angle of internal friction, $\phi$ [degree]                   | 20-37     |

# Table (1) Grain Material Properties <sup>[7]</sup>

## Table (2) Some Input Data

| Symbols | Value | Function  |
|---------|-------|---|
| DSTO    | 9     | Unit weight of stored materials(kN/m <sup>3</sup> ) |
| ALFA    | 45    | Angle of inclination hopper (degree)                |
| Н       | 35    | Height of silo (above conical hopper) (m)           |
| HH      | 7     | Height of conical hopper (m)                        |
| DEM     | 11    | Diameter of silo (m)                                |
| SYS     | 345   | Yield of steel strength (Mpa)                       |
| CS      | 27.6  | Concrete compressive strength (Mpa)                 |
| FAY1    | 20    | Angle of internal friction (degree)                 |
| MUO     | 0.29  | Coefficient of friction                             |
| Dcon    | 24    | Concrete unit weight(kN/m <sup>3</sup> )            |

 Table (3) Some Results of (silo .For)

|                  | Silo.For | Ref. <sup>[8]</sup> | Ref. <sup>[7]</sup> |
|------------------|----------|---------------------|---------------------|
| Max. crack width | 0.244993 | 0.25                | 0.25                |

|                               | U           | · -          |             |
|-------------------------------|-------------|--------------|-------------|
| Variables                     | First trial | Second trial | Third trial |
| Cost (U.P.)                   | 221428      | 264118       | 280982      |
| $\gamma$ (kN/m <sup>3</sup> ) | 8.95        | 6.95         | 7.62        |
| $\alpha$ (degree)             | 42.24       | 49.45        | 57.00       |
| H (m)                         | 32.24       | 29.45        | 37.00       |
| HH (m)                        | 4.24        | 7.45         | 6.00        |
| D (m)                         | 8.24        | 9.45         | 9.00        |
| Max. crack width (mm)         | 0.24499     | 0.24492      | 0.24498     |
| Nexw *                        | 368         | 160          | 384         |

Table (4) The Design Results (initial trial point)

\* Number of (re-design) iteration.

| Table (5) | The Design | <b>Results for</b> | different | angles | of internal |
|-----------|------------|--------------------|-----------|--------|-------------|
|           | frictio    | n of the stor      | red mater | ial.   |             |

| Variables                     | φ=20°   | φ=25°   | φ=30°   | φ=35°   | φ=37°   |
|-------------------------------|---------|---------|---------|---------|---------|
| Cost (U.P.)                   | 221428  | 220415  | 228655  | 236284  | 244402  |
| $\gamma$ (kN/m <sup>3</sup> ) | 8.95    | 8.92    | 9.00    | 8.88    | 8.76    |
| $\alpha$ (degree)             | 42.24   | 42.24   | 42.47   | 42.69   | 42.90   |
| H (m)                         | 32.24   | 32.24   | 32.47   | 32.69   | 32.90   |
| HH (m)                        | 4.24    | 4.24    | 4.47    | 4.69    | 4.90    |
| D (m)                         | 8.24    | 8.24    | 8.47    | 8.69    | 8.90    |
| Max. crack<br>Width (mm)      | 0.24499 | 0.24499 | 0.24498 | 0.24497 | 0.24496 |
| Nexw                          | 368     | 368     | 352     | 336     | 320     |

|                               |         | 1       | 1       |         |         |
|-------------------------------|---------|---------|---------|---------|---------|
| Variables                     | μ=0.29  | μ=0.33  | μ=0.38  | μ=0.42  | μ=0.47  |
| Cost (U.P.)                   | 221428  | 239400  | 294267  | 311133  | 340249  |
| $\gamma$ (kN/m <sup>3</sup> ) | 8.95    | 8.82    | 9.00    | 8.92    | 9.00    |
| $\alpha$ (degree)             | 42.24   | 42.69   | 43.29   | 43.64   | 44.22   |
| H (m)                         | 32.24   | 32.69   | 33.29   | 33.64   | 34.22   |
| HH (m)                        | 4.24    | 4.69    | 5.29    | 5.64    | 6.22    |
| D (m)                         | 8.24    | 8.69    | 9.29    | 9.64    | 10.22   |
| Max. crack                    | 0.24499 | 0.24499 | 0.24499 | 0.24499 | 0.24499 |
| Width (mm)                    |         |         |         |         |         |
| Nexw                          | 368     | 336     | 288     | 256     | 192     |

| Table (6) The Design Results for different coefficients or | f |
|--|---|
| friction between stored material and wall.                 |   |

| I abie (77 I he Design Results for uniterent numbers of | Table (7) | The Design | <b>Results for</b> | different | numbers | of |
|---|-----------|------------|--------------------|-----------|---------|----|
|---|-----------|------------|--------------------|-----------|---------|----|

| Variables                     | N**=4   | N=6     | N=8     | N=10    | N=12    |
|-------------------------------|---------|---------|---------|---------|---------|
| Cost (U.P.)                   | 219361  | 220395  | 221428  | 222462  | 23495   |
| $\gamma$ (kN/m <sup>3</sup> ) | 8.95    | 8.95    | 8.95    | 8.95    | 8.95    |
| $\alpha$ (degree)             | 42.24   | 42.24   | 42.24   | 42.24   | 42.24   |
| H (m)                         | 32.24   | 32.24   | 32.24   | 32.24   | 32.24   |
| HH (m)                        | 4.24    | 4.24    | 4.23    | 4.22    | 4.20    |
| D (m)                         | 8.24    | 8.22    | 8.20    | 8.20    | 8.20    |
| Max. crack<br>Width (mm)      | 0.24499 | 0.24499 | 0.24499 | 0.24499 | 0.24499 |
| Nexw                          | 368     | 368     | 368     | 368     | 368     |

## columns supporting the hopper.

\*\* Columns number



Figure (1) Grain elevator-group





Figure (3) Horizontal and Vertical Pressure on Walls



Figure (4) Axial Force Used in the design



**Figure (5) Forces in Conical hoppers** 



Fig (6) Typical details of hopper supporting beam







Figure (8) Structural analysis of ring beam



Figure(9) Ring beam cross section



Figure (10) Basic data of the numerical example





the minimum cost



Fig (13) Convergence towards the minimum cost



the minimum cost



Fig(14)Minimum total cost vs. angle of internal friction



Fig (15) Optimum unit weight vs. angle of internal friction

Fig(16)Optimum hopper angle vs. angle of internal friction







Fig(18) Optimum hopper height vs. angle of internal friction















Fig (26) Minimum total cost vs. No. of columns support hopper







تقدير الكلفة التصميمية الأقل لخزان (سايلو) الحبوب الدائري

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الخلاصة

تم دراسة تطبيق الطريقة المثلى على مسألة التصميم الإنشائي لخزان (سايلو) الحبوب الدائري ، باعتبار الكلفة الكلية للخزان كدالة هدف وبعض الخواص الهندسية (مثل كثافة المواد المخزونة وزاوية ميلان جدار المفرغ و ارتفاع الخزان وارتفاع المفرغ وقطر الخزان) كمتغيرات تصميميه. تم كتابة برنامج حاسبة لحل الأمثلة العددية بالاستناد إلى معادلات مواصفات المعهد الأمريكي للخرسانة ومتطلبات ومعايير التصاميم الخرسانية.

لقد برهن بان الكلفة الكلية للخزان تزداد بزيادة زاوية الاحتكاك الداخلي بين المواد المخزونة ومعامل الاحتكاك بين المواد المخزونة و الخرسانة وعدد الأعمدة الساندة للمفرغ.

> الكلمات الدالة أمثلية، التصميم الانشائي، الدليل ACI