### AGE-ADJUSTED EFFECTIVE MODULUS MODEL TO EVALUATE LONG TERM EFFECTS IN STEEL-CONCRETE COMPOSITE BEAMS

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#### ABSTRACT

This paper considers the time-varying behavior of composite steel-concrete beams with rigid or flexible connections under service loads. The age-adjusted effective modulus approach is used with evaluating of the aging coefficient " $\chi$ " in absence of connection and characterized by a theoretical point of view. For a simplified expression to evaluate " $\chi$ " and for an easy method to implement, a closed-form solution in the hypothesis of rigid connections and uncracked concrete is proposed. The method is verifyied by comparing with the results of another finite element model.

#### **KEYWORDS**

Age-adjusted modulus, composite beams, creep, shrinkage, shear connectors.

#### **INTRODUCTION**

Steel-concrete composite beams are popular and have an economical form of construction in both buildings and bridges. Composite beams must satisfy the requirements of both strength and serviceability limit states. To check the latter, a correct assessment of creep and shrinkage effects on stress and deflection response is very important, the assessment carried out generally in the hypothesis of linear-elastic behavior of steel and linear-viscoelastic behavior of concrete. It is pointed out that during the last few years the use of stronger materials has led to an increase in extreme-fiber stain at service load. As a result, deflection limits, mainly introduced to reduce concrete cracking and to preserve supported elements together with serviceability stress limits based on durability criteria, govern the design of composite beams more and more.

Since the closed-form solution available for riged and deformable connections, is referred to as a simplified viscous law, it cannot be adopted in general. Creep and shrinkage laws that follow the actual behavior of concrete (CEB 1984,1990<sup>[7]</sup>;ACI 1992<sup>[1]</sup>) are in fact very complex to be adopted and in any case, the exact solution can be determined only for very simple structural schemes.

For these reasons, in general actual codes make references to simplified algebraic methods for the practical evaluation of the time dependent response. They are the effective modulus (EM) method, the mean stress (MS) method, and the age-adjusted effective modulus (AAEM) method if an accurate evaluation of viscous effects is required.

In this work, initially the aforementioned algebraic methods are characterized. Then, using the AAEM method, the age-adjusted effective modulus of a cross section is assumed as the one computed in the absence of shear connection. A criterion for its theoretical evaluation for creep and relaxation problems is provided. With reference to the CEB 90 code code, a simplified expression to evaluate the age-adjusted effective modulus in a practical way and some useful interpretations on the properties of the proposed method are given.

A set of examples show the capability of obtaining very accurate, long-term solutions for composite beams with rigid or flexible connections. This property is very important since experimental tests have shown that in composite beams the interaction between the concrete slab and the steel beam is never complete.

#### **ALGEBRAIC METHOD**

The difficulties, which are encountered when solving a viscous problem, are due to the fact that a law must be considered for concrete which within the hypothesis of linear viscosity may be presented in the following integral of stress-strain law:

$$\mathcal{E}_{c}(t) = \mathcal{E}_{c}(t,t_{o}) = \int_{t_{o}}^{t} \Phi(t,\tau) d\sigma_{c}(\tau) + \mathcal{E}_{n}(t)$$
(1)

This is overcome by transforming the integral equation into a simple linear algebraic equation. In Eq. (1),  $\varepsilon_c(t)$ = total strain in the concrete at time t;  $\varepsilon_n(t)$  = a stress-independent strain;  $\sigma_c(t)$ = stress that at time t<sub>o</sub> it has a finite value  $\sigma_c(t_o)$  and in the interval (t<sub>o</sub>,t) it varies arbitrarily; and  $\Phi(t,\tau)$  = creep function, usually provided by codes. In particular, the AAEM method, which is the most general and accurate among the algebraic methods, is based on the transformation of the superposition integral in Trost's (1967) algebraic equation is:

$$\int_{t_o}^{t} \Phi(t,\tau) d\sigma_c(\tau) = [\sigma_c(t) - \sigma_c(t_0)] \mu(t,t_o) \Phi(t,t_0)$$
(2)

where  $\mu(t,t_o) = an$  unknown coefficient [related to the unknown function  $\sigma_c(t)$ ].<sup>[2]</sup>

In this way, by introducing the creep coefficient  $\varphi(t,t_o)$  so that:

$$\Phi(t,\tau) = \frac{\mathcal{E}_c(t,\tau)}{\sigma_c(\tau)} = [1 + \varphi(t,\tau)] \frac{1}{E_c(\tau)}$$
(3)

where  $E_c(\tau)$ = Young's modulus of concrete. If the strain  $\varepsilon_c(t)$ - $\varepsilon_n(t)$  produced by the tension  $\sigma_c(t)$  follows the law

$$\varepsilon_{\rm c}(t) - \varepsilon_{\rm n}(t) = \varepsilon_{\rm o+} \varepsilon_1 \varphi(t, t_{\rm o}) \tag{4}$$

with  $\varepsilon_{o}, \varepsilon_{1}$  = arbitrary coefficients, the integral (2) is independent of  $\sigma_{c}(t)$  and  $\mu(t,t_{o})$  can be exactly determined a priori. Indeed, to calculate  $\mu(t,t_{o})$ , by introducing the relaxation function R(t,t\_{o}), it is easy to obtain the equation [Bazant 1972]<sup>[4]</sup>.

$$\mu(t,t_o) = \frac{1}{1 - R(t,t_o) / E(t_0)} - \frac{1}{\Phi(t,t_0) [E(t_0) - R(t,t_o)]}$$
(5)

The condition (4) therefore provides an easy determination of  $\mu$ , once  $\Phi(t,t_o)$  is known and  $R(t,t_o)$  is determined. By this hypothesis, for a given  $\mu$ , the constitutive equation (1) can be exactly transformed into the familiar algebraic equation

$$\mathcal{E}_{c}(t) - \mathcal{E}_{n}(t) = \sigma(t_{o}) \Phi(t, t_{0}) + [\sigma_{c}(t) - \sigma_{c}(t_{o})]\mu(t, t_{o})\Phi(t, t_{0})$$
$$= \frac{\sigma_{c}(t)}{E_{cadj}} + \sigma_{c}(t_{o}) \left(\frac{1}{E_{ceff}} - \frac{1}{E_{cadj}}\right) = \frac{\sigma_{c}(t)}{E_{cadj}} + \frac{-}{\varepsilon}(t)$$
(6)

where  $\overline{\varepsilon}(t) =$  an imposed strain linked to the viscousity effects in the interval  $(t,t_o)$ , while the quantities

$$E_{ceff} = \frac{1}{\Phi(t,t_o)} = \frac{E_c(t_o)}{1 + \varphi(t,t_o)}$$
(7)

$$E_{cadj} = \frac{1}{\mu(t,t_o)\Phi(t,t_o)} = \frac{E_c(t_o)}{1 + \chi(t,t_o)\varphi(t,t_o)}$$
(8)

are the fictitious modulus-effective modulus and age-adjusted effective modulus, respectively-the last one is easily known when the aging coefficient  $\chi(t,t_o)$  is introduced.

By means of (6), it is possible to deduce as, assuming the elastic modulus for the concrete be equal to  $E_{cadj}$  and introducing the imposed strain  $\dot{\epsilon}(t)$ , that the viscous problem becomes a simple elastic equilibrium problem.

The AAEM method or " $\chi$ " method also covers a significant theoritical role since it makes easy to obtain the EM method by assuming  $\chi =1$  so that the material considered elastic with modulus  $E_{ceff}$  ( $\dot{\epsilon}$  is zero). By a theoritical point of view, the approximation adopted in this case is evident too: it consist in calculating the integral of superposition expressed by (2) by the rectangular rule in a single time step. Assuming that modulus of concrete is constant at any time and equal to  $E_c$ , and taking  $\chi = 0.5$ , then the expression is immediately obtained at the base of the mean stress method where the determination of the superposition integral is solved by applying the trapizoidal rule in a single time step.

### AAEM METHOD FOR STEEL-CONCRETE COMPOSITE BEAMS

Following the hypotheses of the AAEM method for homogeneous structures, it is evedent that by adopting the  $\chi$  or  $\mu$ values determined in line with (4), an approximate solution is determined for composite beams.

The nonhomogeneity, due to the presence of a viscous material (concrete) and an elastic material (steel), involves a migration of stresses from a point of the structure to another with variation laws that do not respect the equation (4). Due to this reason, Trost (1967) proposed two  $\chi$  coefficients in the presence of rigid connections: the former,  $\chi N$ , related to the normal force in the concrete component beam; the latter,  $\chi M$ , related to the bending moment in the same beam. In general, these two coefficients are not easily determinable. In the hypothesis of the strong steel beam with respect to the concrete slab as well as the affinity of the shrinkage law with the creep law, for creep or relaxation problems, Trost found simple relations to evaluate  $\chi_N$  in rigorous way and  $\chi_M$  in an approximate way.

The complexity of the problem due to the presence of two coefficients  $\chi$  as well as the limits due to the aforementioned hypotheses induced (Amadio 1993)<sup>[2]</sup> to to propose the use of an approximate  $\chi$  value only for the composite beams with rigid or deformable connections. For three elementary problems of creep, relaxation and shrinkage, these  $\chi$  values were evaluated in the hypothesis of no connection between the concrete and steel beam and collected in tabilar form for practical use.

Afterwards, in this paper, an exact theoretical formulation of this  $\chi$  coefficient for creep and relaxation problems is provided together with some practical expressions that supply  $\chi$  values for these two problems as well as shrinkage.

## χ VALUES FOR A CREEP PROBLEM (SUSTAINED LOAD)

In order to discuss the issues reported here, an infinitesimal element of a composite beam has to be considered without shear connection (Fig.1), and subjected to a constant bending moment Mo(x)in time. The concrete beam has area Ac, moment of inertia Ic, Young's modulus Ec(t) and a viscoelastic behavior, while the steel beam has area As, moment of inertia Is, Young's modulus Es and a linear-elastic behavior.

In this case, each component beams works in parallel and due to creep effects in the concrete, at every time t, the bending moment in the beams varies, but owing to equilibrium and compatibility conditions, the stresses in every point of the slab follow the same law of variation. Therefore, the structure admits only one value  $\chi = \chi_M = \chi_c(t,to)$  for the section. Suposing  $\chi_c(t,to)$  is known, and using (6) it is possible to obtain the constitutive equations for the sections at time to and time t, according to the following form:

$$\theta_c(t_o) = -\frac{M_c(t_o)}{E_c(t_o)I_c} \qquad \theta_s(t) = -\frac{M_s(t)}{E_sI_s}$$
(9 a,b)

$$\theta_c(t_o) = -\frac{M_c(t)}{I_c E_{cadj}} - \frac{M_c(t_o)}{I_c E_{cadj}} \left(\frac{1}{E_{ceff}} - \frac{1}{E_{cadjf}}\right) \qquad \qquad \theta_s(t_o) = -\frac{M_s(t_o)}{E_s I_s}$$
(10 a,b)

where  $\theta c(t)$ ,  $\theta s(t)$ , Mc(t), Ms(t), and  $\theta c(to)$ ,  $\theta s(to)$ , Mc(to), Ms(to) = curvatures and bending moments in the beams at long-term and initial time to, respectively.

To characterize this solution, it is sufficient to consider both compatibility and equilibrium conditions at a time t :

$$\theta_c(t) - \theta_c(t_o) = \theta_{cr}(t) = \theta_{sr}(t) = \theta_s(t) - \theta_s(t_o)$$
(11)

$$M_{c}(t) - M_{c}(t_{o}) = M_{cr}(t) = M_{sr}(t) = -[M_{s}(t) - M_{s}(t_{o})]$$
(12)

Following Trost's (1967) approach, by means of (7)-(12), the following equation is immediately obtained:

$$M_{cr}\left[\frac{E_{c}(t_{0})J_{c}+E_{s}J_{s}}{E_{s}J_{s}}+\chi(t,t_{0})\varphi(t,t_{0})\right]=M_{c}(t_{0})\varphi(t,t_{0})$$
(13)

By introducing the stiffness ratio  $\boldsymbol{\beta}$  , that characterizes the composite section:

$$\beta = -\frac{E_s J_s}{E_c(t_o) J_c + E_s J_s} \tag{14}$$

Eq.(13) can be stated in the following form:

$$M_{cr}(t) = -\frac{\beta \ \varphi(t,t_0)}{1 + \chi(t,t_0)\varphi(t,t_0)} M_c(t_o)$$
(15)

Furthermore, by setting  $\sigma_c(t)$ -  $\sigma_c(t_o)$ =  $\sigma_{cr}(t)$  in (6), it is evident that in the absence of inelastic strain the following is obtained:

$$\sigma_{cr}(t) = -\frac{\varphi(t,t_0)}{1+\chi(t,t_0)\varphi(t,t_0)} \left[ \sigma_{cr}(t_0) - E_c(t_0) \frac{\mathcal{E}_c(t) - \mathcal{E}_c(t_0)}{\varphi(t,t_0)} \right]$$
(16)

By comparing (15) and (16), it is clear that the stress evolution in the composite beam for the creep problem is the same as for a problem of pure relaxation [ $\varepsilon_c(t)$ -  $\varepsilon_c(t_o) = 0$ ] for a homogeneous concrete structure where the creep coefficient is assumed to be:

$$\varphi(t,t_0) = \beta \varphi(t,t_0) \tag{17}$$

Then, it is possible to compute  $\sigma_c(t,t_o)=\dot{R}(t,t_o)$ , where  $\dot{R}$  is the relaxation function obtained by using  $\phi$  as the creep coefficient. Next, by directly using (2) and (8), it is possible to compute the parameters  $\mu$  and  $\chi = \chi_M$  of the actual problem. Eq.(5 ) cannot be used in this case because the actual problem is not a relaxation problem and therefore the equality (4) is not satisfied.

In the hypotheses of null connection, a few additional dificulties with respect to a homogeneous structure are introduced. By observing (15) it is evident that varying the coefficient  $\chi$ , in any case, results in solutions that respect the equilibrium and compatibility conditions. By adopting the EM or

the MS method or using the hypothesis that strain in the concrete slab follows (4), the actual response that is strickly related to the characteristics of steel beams is disregarded. Only by following the proposed approach, it is possible to evaluate correctly the behavior of the composite beam without connection.

Obviously, the method becomes approximate in the presence of an elastic or rigid connection. For rigid connections, in particular,  $\chi$  is not constant in the section but varies along the fiber following the law<sup>[2]</sup>:

$$\chi(y) = \frac{\Delta\sigma_N}{\Delta\sigma_N + \Delta\sigma_M y/(h/2)} \chi_N + \frac{\Delta\sigma_M y/(h/2)}{\Delta\sigma_N + \Delta\sigma_M y/(h/2)} \chi_M$$
(18)

where y= coordinate referred to central fiber; h= thickness of the concrete slab;  $\Delta \sigma_M$  and  $\Delta \sigma_N$  = aging coefficients of the section due to the same forces.

The problem appears more complicated for deformable connections, since, in general, we cannot work in a cross section, and so  $\chi_M$  and  $\chi_N$  are related to the overall beam respons. However, these variations along the beam are limited (in general) and an approximate treatment is still possible.

To understand the properties of the solution provided by the proposed  $\chi$  values reference is made to the composite beam theory developed by Newmark et al (1951), which in the presence of deformable connections gives good results if compared with the experimental response (Wright 1990)<sup>[8]</sup>. For this beam model (Fig.2), the assumption that the slip-shear flow law is linear and elastic is obviously a simplification since the slip develops only when bond and friction strengths are exceeded and the connectors are activated. On the other hand, this is the more simple approach acceptable for this type of beam.

Afterwards, a connection rigidity K(x) constant along the beam is assumed since  $\chi$  is not influenced practically by this parameter. Therefore, the solution determined without connection can be interpreted as a limit condition when  $K\rightarrow 0$ . It is also easy to observe that from both a numerical and theoretical viewpoint, It is assumed that  $\chi = \chi_M$  and simultaneously  $\chi_N = \chi_M$ , by adopting the  $\chi$  values determined without connection.

The aging coefficient  $\chi_N$  is instead characterized by strong variations. In particular, the asymptotic behavior for  $\chi_N$  occurs when the initial normal force is almost coincident with the final normal force and the superposition integral area is different from zero. In this case, in Eq. (2) the first member is different from zero, but at the second member the quantity  $\sigma_c(t) - \sigma_c(t_o)$  vanishes and therefore  $\mu(t,to)\Phi(t,to)$  and  $\chi(t,to)$  aim at infinite [see Eq. (8)].

To use the AAEM method with the same simplicity as for the EM or MS methods, it is important to provide a simple relation in order to calculate the  $\chi$  coefficient.

In particular, it is important that the value of  $\chi = \chi_c(t,t_{\infty}) = \chi_{c\infty}$ , linked with the long term solution, is determined. To this aim , with references to the CEB Model Code 90, an exact solution of  $\chi_{c\infty}$  (continum line of Fig.3) is presented together with an extension of the approximate expression given by Lacidogna (1993)<sup>[7]</sup>, in which the aging coefficient was determined under

the assumption that the strain satisfies the relation (4) and the structure is homogeneous.

By using the continuum lines of Fig.3, it is possible to perform a linear interpolation to determine  $\chi_{c\infty}$  values. By using the Lacidogna approach<sup>[7]</sup>, an expression for  $\chi$  is suggested:

$$\chi_{c\infty} = \chi_c (3.10^4) = \frac{t_o^{0.5}}{n + t_o^{0.5}}$$
(19)

where n = corrective coefficient calibrated on the fictitiousthickness  $h_0=2A_c/u$  (in cm) ( $A_c$  is the area of the concrete section while u represents the perimeter in contact with the atmosphere), the relative humidity (RH) (%), the characteristic strength of concrete  $f_{ck}$  (MPa), and the coefficient  $\alpha$  are needed. The coefficient n is calculated as summation of Lacidogna's (1993) term  $n_L$  and the corrective term  $n_c$ . The values of  $n_L$  and  $n_c$  are defined as:

$$n_{L} = f_{a}(h_{o}) \left[ 1 + \left( 1 - \frac{RH}{50} \right) f_{b}(h_{o}) \right] f_{c}(f_{ck})$$
(20)
with:

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$$f_a(h_o) = \frac{0.28 h_o^{1/3}}{e^{(10^{-3}h_o)}}$$
(21a)

$$f_b(h_o) = -0.772 + 2.917. \ 10^{-3}h_o \tag{21b}$$

$$f_c(f_{ck}) = 0.772 + 0.0114 f_{ck}$$
(21c)

and

$$n_c(\alpha, h_o) = \frac{(1-\alpha)^3}{2.5} + (\frac{1}{4} + \frac{h_o}{100} - \frac{{h_o}^2}{23500})(1-\alpha)$$
(21d)

These equations provide accurate results when  $5 \le h_o \le 160$  cm,  $50\% \le RH \le 80\%$  and  $3 \le t_o \le 200$  days and imply 5% maximum error and 1% medium error. In Fig. 3, a comparison between the exact and the approximate (dotted line)  $\chi_{c\infty}$  values is reported also. Even though the  $\chi$  values obtained by (19) encompass a maximum error of 5%, the beam responses obtained by these previous approximate or exact  $\chi$  values are practically coincident.

# $\boldsymbol{\chi}$ VALUES FOR RELAXATION PROBLEMS (IMPOSED DISTORTION)

The condition of imposed flexural distortion, i.e., the relaxation problem, is important to evaluate the stress state in a statically indeterminate composite beam with constant mechanics characteristics subjected to a settlement of the supports. In this case it is possible to set:

$$-\frac{M_{s}(t_{o})}{I_{s}E_{s}} = -\frac{M_{c}(t_{o})}{I_{c}E_{co}} = -\frac{M_{s}(t)}{I_{s}E_{s}} = \theta_{o}$$
(22)

to evaluate the effect of an imposed curvature,  $\theta_0$ , constant in time. By means of (10a) and (22), the following is obtained:

$$M_{c}(t) = -I_{c}\theta_{o} \left( E_{cadj} - \frac{E_{cadj} - E_{ceff}}{E_{ceff}} E_{co} \right)$$
(23)

As a result, it is evident that the concrete beam is subjected to an effect of pure relaxation that does not depend on the steel beam. This allows affirmation of the fact that the  $\chi$  coefficient determined by means of the proposed approach is equal to that of homogeneous structures subjected to a constant strain and vice versa.

Analogous results are obtained by assuming rigid connections, since even in this case the strain law (4) adopted for the determination of the  $\chi$  coefficient for homogeneous structures is exactly respected with reference to concrete, i.e.,  $\varepsilon_c$  is constant in time in accordance with the theorm of the linear viscoelasticity.

For a practical application,  $\chi_{\infty}$  values, denoted here as  $\chi_{r\infty}$ , can be easily found by using (21d) setting  $\alpha = 1$ .

#### χ VALUES FOR SHRINKAGE PROBLEMS

To evaluate slab shrinkage effects it should be observed that the laws of shrinkage evolution in time, proposed by actual codes, are not affine with the creep laws in general (if shrinkage is affine to creep,  $\chi$  shrinkage values can be calculated as in the case of constant load, by assuming  $\chi_s = \chi_c$ ). In this case, however, a numerical analysis for a composite beam without connections can be performed. Clearly, since the solution without connections is characterized by purely deformative effects of the slab, the  $\chi$ values have to be determined for a beam with elastic connection in the limit condition of K $\rightarrow$ 0. It is underlined that for shrinkage problem, when K $\rightarrow$ 0, in general  $\chi_M$  is not equal to  $\chi_N$ ; they are equal when the steel beam stiffness is low compared to the concrete stiffness.

Also for the shrinkage problem, by using CEB Model Code 90, an approximate expression for the  $\chi_s(t_{\infty}, t_o) = \chi_{s\infty}$  coefficient is

proposed in this work to evaluate the long-term effects. For this model the significant parameters are the initial time load (at  $t_o$ ), the *RH*, the characteristic strength of concrete  $f_{ck}$ , and the fictitious thickness of the slab  $h_o$ . The stiffness of the steel beam or concrete slab does not influence the response practically. The  $\chi$  values, determined with reference to a section in which  $\chi_M = \chi_N$ , i.e., for a small stiffness of the steel beam,  $\chi_{s\infty} = \chi_s(3.10^4, t_o)$  becomes

$$\chi_{s\infty} = -\frac{(RH - 75)}{1800} - (\frac{f_{ck} - 30}{1120}) + \left[\frac{1}{5} + \frac{1}{2.5 + 0.25} \frac{h_o}{h_o} - \frac{51}{(9.6 + h_o)^2}\right] \log_{10}(t_0) + \frac{5.2}{h_o} - \frac{9.76}{h_o^2}$$
(24)

Using the same limits of application, as for  $\chi_{c\infty}$ , for  $\chi_{s\infty}$  values the average percentage error and the maximum absolute error are about 1.44% and 3.3%, respectively. Fig.4 shows a comparison between numerical (continuum line) and the approximate (dotted line)  $\chi_{s\infty}$  values.

#### **Method Application**

As an application example, the statically determinate beam of Fig.5 is considered. It is subjected to a uniform load or to the effect of the shrinkage of the slab. For this beam, a comparison can be made between the proposed  $\chi$  method [or AAEM method] and the numerical response obtained by [Amadio and Fragiacomo (1993a)]<sup>[3]</sup> which were obtained by a finite element

model incorporating a general law of viscosity to describe the response of such structure under the long term effects.

The comparison is made for the section of Fig.5. By varying the rigidity K in the connection (the current rigidities are generally included between 104-106 N/cm<sup>2</sup>), hypothesizing a relative humidity to 55% and a concrete with mean compressive strength  $f_{cm}$ =3800 N/cm<sup>2</sup>.

For the uniform load condition, taking  $t_0=10$  days, Figs.6,7,8 and 9 show the ratios of the  $\eta_{\infty}$  deflection, the bending moment  $M_{c\infty}$  of the concrete, the bending moment  $M_{s\infty}$  of the steel, and the normal force  $Nc_{\infty}$  in the mid-span cross section after 100 year (which can be virtually taken as infinite time) respectively, as well as the corresponding initial elastic values  $\eta_{o}$ ,  $M_{co}$ ,  $M_{so}$  and  $N_{co}$ .

If the figures are examined, it can be seen that even in such a simple scheme the correct solution obtained numerically <sup>[3]</sup> is close to the result that is obtained by using the proposed  $\chi$  method.

Figures 10,11,12 and 13 show the effects on the deformation and stress state of the beam caused by shrinkage in the slab, if to=10 days and RH=75% are taken. The y-coordinate containes the  $\eta_{\infty}$  /  $\eta$ cr the deflection in the mid-span at infinitive time, and the Mc<sub> $\infty$ </sub>/Mcr, Ms<sub> $\infty$ </sub>/Msr, Nc<sub> $\infty$ </sub>/Ncr ratios between the bending moment values and the normal force at infinite time with the respective cross section strength values.

In this case it can be seen how the proposed method are in general supplies a reasonable solution.

From a parametric analysis carried out, it was able to confirm that even for more important sections (bridge cross sections, for example) the considerations set out above remain valid.

From the examples described, it can be seen how the proposed formulation may be extremely useful to the designer for the study of statically determinate structures, especially when they are of a certain importance and therefore require accurate assessment of the creep response.

#### CONCLUSIONS

The simplified method described in this work, which depends on the age-adjusted effective modulus, presents a simple solution of viscous problems in steel-concrete composite beams with rigid or deformable connections when concrete can be considered uncracked. The theoretical characterization of the proposed  $\chi$  values and their determination by means of simplified expressions permits, in fact, a simple use for the designer.

The proposed approach used in the presence of rigid connection and a constant cross section allows both a correct interpretation of the viscous problem and a solution characterized by a high precision and the same difficulties as for the EM method. The comparison with the solutions obtained by a numerical method <sup>[3]</sup> allows emphasis on the advantage of the proposed approach, in particular for shrinkage problem.

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#### **NOTATION**

The following notation are used in this paper

A= beam cross section area;

E= Young's modulus;

I= moment of inertia;

L= length of the beam;

M= Bending moment force;

N= normal force;

*n*= modulus ratio;

R= relaxation function;

S= shear force in concrete;

t, $\tau$ = time;

 $\eta$ = vertical deflection;

 $\theta$ = beam curvature;

 $\sigma,\epsilon$ = stress and strain;

 $\Phi, \phi$ = creep function and creep coefficient; and

 $\chi_c$ ,  $\chi_r$ ,  $\chi_s$  = aging coefficient for creep, relaxation, and shrinkage problems.

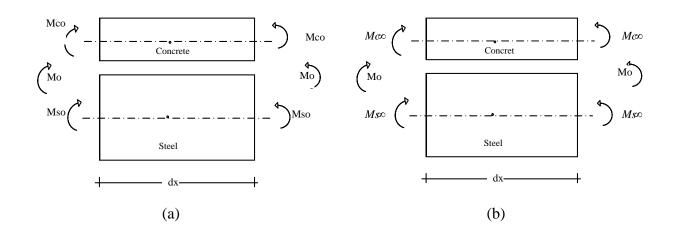
#### **Subscripts**

- c = concrete;
- s = steel;
- o = initial time;
- $\infty =$ infinite time.

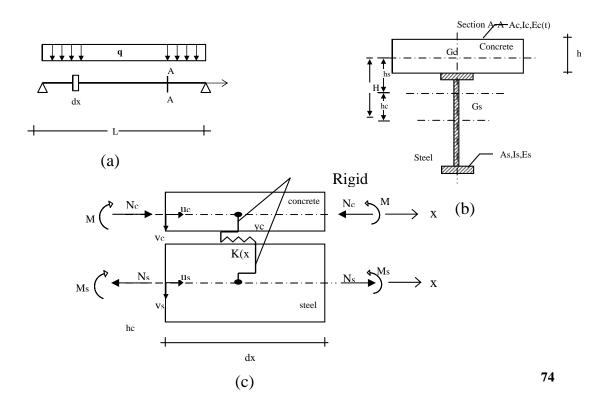
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**Fig. 1**. Composite Beam without Connection Subjected to constant Bending Moment: (a) Initial Time (b) Infinite Time



**Fig. 2** Beam with Elastic Connections: (a) Beam Model; (b) Cross Section; (c) Detail of the Connection.

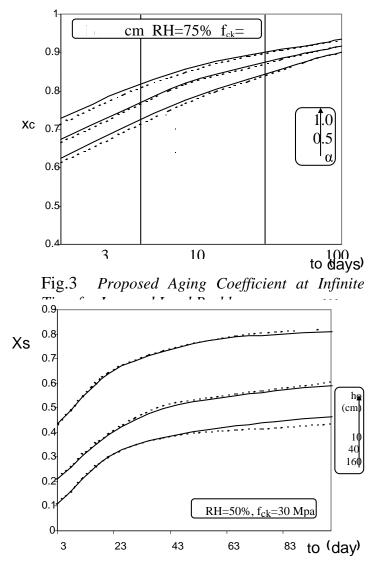


Fig.4 Proposed Aging Coefficient at Infinite Time for Shrinkage Problem

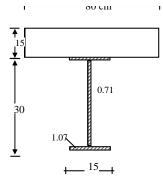


Fig. 5 Beam Cross Section (Typical of Commercial Building)

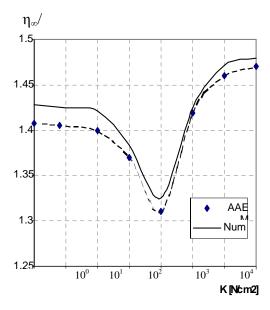


Fig.6 Response in Terms of Displacement at Applied Load

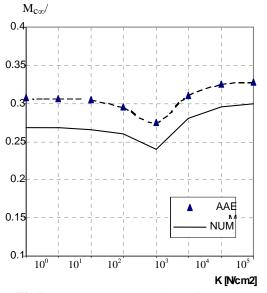
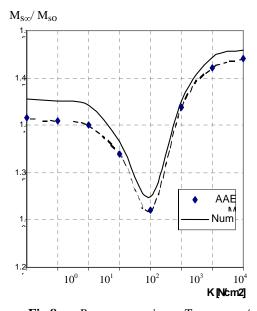
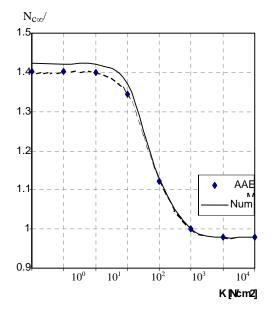


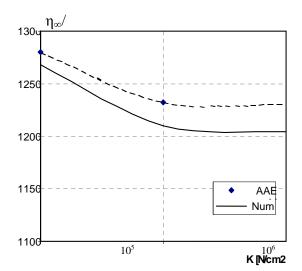
Fig.7 Response in Terms of Bending Moment in the Slab at Applied Load



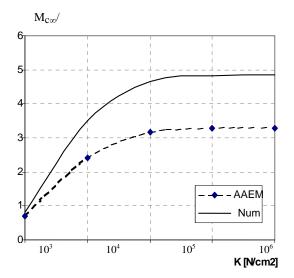
**Fig.8** Response in Terms of Displacement Moment in the Slab at Applied Load



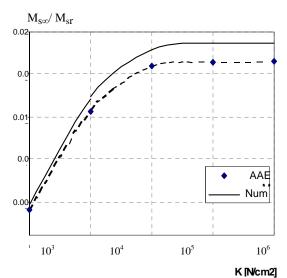
**Fig.9** Response in Terms of Normal Force in the Slab at Applied Load



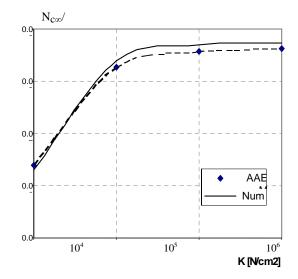
**Fig.10** Response in Terms of Displacement at the Effects of Shrinkage in Slab



**Fig.11** Response in Terms of Bending Moment in the Slab at the Effects of the Shrinkage



**Fig.12** Response in Terms of Normal Forces at the Effects of Shrinkage in Slab



**Fig.13** Response in Terms of Bending Moment in Steel at the Effects of Shrinkage

(قسم الهندسة المدنية، كلية الهندسة، جامعة تكريت)

#### الخلاصة

تم في هذا البحث تطوير طريقة مبسطة لحساب تأثيرات الزحف والانكماش في العتبات المركبة والتي تحتوي على روابط مقاومة القص الصلدة أو القابلة للتـشوه وذلك باستخدام طريقة معامل التقدم الزمني χ ( Aging coefficient)، حيث ---- تم حساب صيغ مبسطة لمعامل التقدم الزمني بالاعتماد على قيم سابقة محـسوبة على أساس عدم وجود ربط بين خرسانة السقف وحديد العتبة. لغرض صياغة معامل التقدم الزمني المستخدم في نموذج التحليل تـم الاعتماد لغرض صياغة معامل التقدم الزمني المستخدم في نموذج التحليل تـم الاعتماد يصف تصرف الخرسانة عند إدخال عامل الزمن (Closed form solution) القانون اللزوجة العام الذي يصف تصرف الخرسانة عند إدخال عامل الزمن (Long-term behavior). قابلية النموذج على التنبؤ بتصرف العتبة المركبة تم التحقق منها من خلال مقارنـة النتائج التي تم الحصول عليها لأحد المقاطع التي تم تحليلها مع النتائج النظرية لأحد النماذج العددية التي تعتمد على طريقة العناصر المحددة، كانت النتائج مقاربة بشكل النماذج العددية التي تعتمد على طريقة العناصر المحددة، كانت النتائج مقاربة بشكل مام.

#### الكلمات الدالة

المنشآت المركبة، الزحف، الانكماش، روابط القص، معامل التقدم الزمني.