EFFECT OF BOUNDARY CONDITIONS ON IMPACT STRESSES OF BEAMS

Dr. Eyad K. S. Al- Hachamee.
Department of Building and Construction
The University of the Technology

ABSTRACT

A theoretical analysis based on the numerical solution of the beam impact integral equation is carried out to determine the impact force and deflection time histories, the strain energy absorbed by the beams and the maximum bending moment.

Effect of beam boundary conditions on impact response of beam is also discussed. The theoretical results obtained in the present analysis are compared with experimental and theoretical works previously done. A good agreement is found between theoretical and experimental results. This indicates that the impact resistance of relatively large beams may be predicted by using the theoretical approach based on equation of undamped beam vibration. All the derivations required to predict the effect of boundary conditions are performed for both forced and free vibrations. For the same falling mass and the same applied kinetic energy (height of drop) for all cases, the maximum central deflection and the maximum impact force are affected by the boundary conditions of the beams

KEYWORDS

Beam, Boundary Conditions, Deflection – Time Histories, Force – Time Histories, Free and Forced Vibration, Impact, Striker.

LIST OF NOTATION

a Relative approach of striking bodies.

E Young modulus of elasticity.

F Impact force.

I Moment of inertia.

k Hertz (Deformation) constant.

K E Kinetic energy.

L Span.

m Mass per unit length of the beam.

m_b Mass of the beam.

m_s Mass of the striker..

t Time.

U Strain energy.

V_o Velocity of striker at instant of impact.

Y Deflection.

Y_b Central deflection of the beam.

Y_s Displacement of the striker.

τ Impact duration.

ω Angular frequency

INTRODUCTION

Impact loads may be applied to many structures which have been designed only to resist their own dead loads in addition to the conventional static live loads. If the probability of impact loading is very small, it may be uneconomical to design against impact loads, but if the structure is subjected to impact the results could be very serious. Under these circumstances, it is useful to check the impact resistance of structures which have been designed to resist static loads. Some structures such as shelters and buildings of nuclear plant must be designed to resist impact loads. Missile impact, fragments impact, ship collision, vehicle impact with structures, and falling masses in industrial buildings are some examples of impact.

Local response and overall (structural) response are usually associated with impact. The structural responses are in the form of flexural and shear deformations, and the structure is to be dynamically analyzed under the applied force-time history. The effect of impact loading on concrete structures has received a considerable amount of attention of many researchers [1-8].

The objective of this study is to present a theoretical analysis based on the numerical solution of the beam impact integral equation. Based on equation of undamped beam vibration, the effect of boundary conditions on the beam response to impact force is also presented for both stages of vibration (forced and free vibrations).

The impact force and deflection-time histories, the strain energy absorbed by the beam, and the bending moment are all determined.

IMPACT INTEGRAL EQUATION

The structural dynamic response of structures subjected to impact can be determined if the impact force time history is known. Therefore the main purpose of the impact analysis is to determine the impact force-time history F(t), deflection Y (x, t). A beam is struck transversely by a mass (ms) having a spherical surface at the point of contact and striking velocity (V_o), figure (1).

The formulation of this problem can be effected only under certain assumptions:-

a) All assumptions of the classical theory of beam are applicable.

b) Hertz law of impact is valid [9], hence

c)

$$F(t) = k \cdot (a(t))^{3/2} \qquad \text{---------- (1)}$$

Where:

F (t): the impact force at any time (t) within the duration of impact.

a (t): the relative approach of striking bodies. Figure (1)

k : the hertz (deformation) constant which depends on the elastic mechanicalproperties of two bodies and on the shape of the two bodies at the contact zone [10].

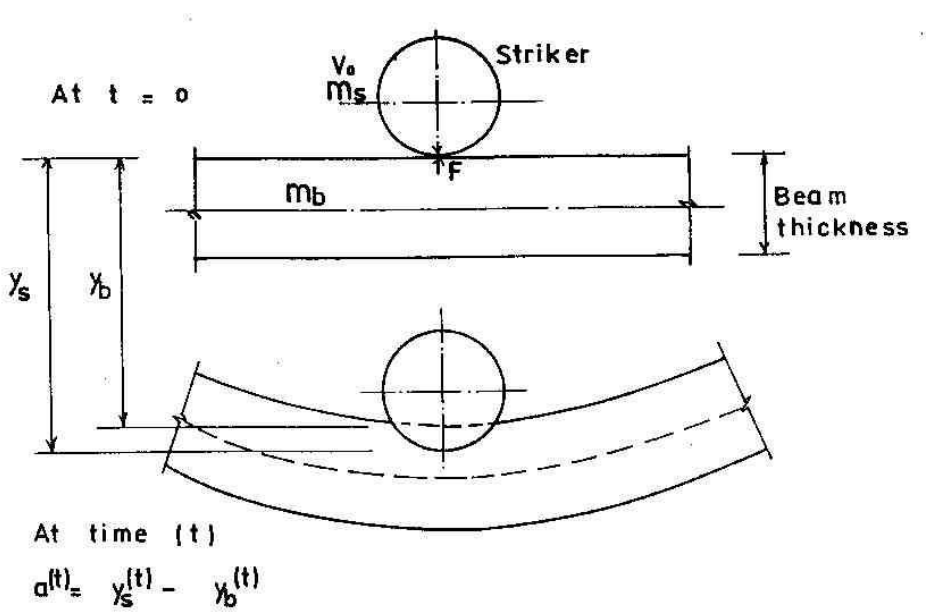


Fig.(1) Displacement at Impact Zone

The deformation equation is:-

$$a(t) = Y_s(t) - Y_b(t) \qquad \text{---------- (2)}$$

Where:

$Y_s(t)$: The displacement of the rigid striker under action of the force F (t).

$Y_b(t)$: The deflection of the beam at the point of contact.

Here $Y_S(t)$ is given by [11, 12, 13, 14, 15]

$$Y_s(t) = V_o \cdot t - \frac{1}{ms} \int_o^t d\tau \int_o^\tau F(\bar{\tau})\ d\bar{\tau} \qquad \text{---------- (3)}$$

Where:

ms: mass of the striker.

V_o: initial velocity of striker (impact velocity).

Substituting equation (3) into equation (2) and making use of equation (1)

$$\left(\frac{F(t)}{k}\right)^{2/3} = V_o \,.\, t - \frac{1}{ms}\int_o^t d\tau \int_o^\tau F(\bar{\tau})\; d\bar{\tau} - Y_b(t) \qquad \text{---------- (4)}$$

The well-known equation of undamped beam vibration is [11, 14, 16, 17,]

$$EI\,\frac{\partial^4 Y}{\partial x^4} + m\,.\,\ddot{Y} = P(x,t) \qquad \text{---------- (5)}$$

Where:

EI: the rigidity of the beam.

Y : the displacement of the beam which is a function of the time (t) and position (x)

m : the beam mass per unit length.

P (x, t): the external load intensity.

This equation can be solved for Y (x,t), the beam deflection as a function of both time and position.

Free Vibration

For the free beam vibration, P (x,t) = o, the beam displacement $Y_b(x,t)$ can be represented by:-

$$Y_b(x,t) = \sum_{i=1}^{\infty} Y_i = \sum_{i=1}^{\infty} X_i(x)\;.T_i(t) \qquad \text{---------- (6)}$$

Where:

$X_i(x)$: The characteristic shape function.

$T_i(t)$: The time function.

The substitution of this assumed displacement in equation (5) gives:-

$$\frac{EI}{m}\cdot\frac{X_i'''}{X_i} = -\frac{T_i''}{T_i} = +\omega_i^2 \qquad \text{---------- (7)}$$

Where:

ω_i: is an arbitrary constant for (ω_i > o) the shape and time function will be [11, 14, 15, 17].

$X_i(x) = A_i \sin a_i x + B_i \cos a_i x + C_i \sinh a_i x + D_i \cosh a_i x$

$T_i(t) = E_i \sin \omega_i t + F_i \cos \omega_i t$

Where:

$$a_i = \sqrt[4]{\frac{m\omega_i^2}{EI}} \qquad \text{---------- (8)}$$

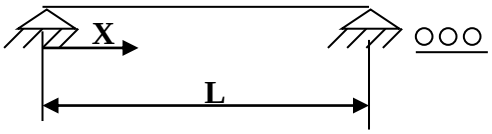
So the displacement Yi (x,t) for ith mode will be

$Yi(x,t) = (A_i \sin a_i x + B_i \cos a_i x + C_i \sinh a_i x + D_i \cosh a_i x)(E_i \sin \omega_i t + F_i \cos \omega_i t)$

The constant A_i, B_i, C_i and D_i are determined from the beam boundary conditions, while E_i, F_i are determined from the beam initial conditions.

Simply Supported Beam

Boundary conditions



Y_i (x,t) = o and $\frac{\partial^2 Y_i}{\partial x^2} = o$ at x = o, L

Therefore:

$B_i = C_i = D_i = o$

$$\omega_i = \frac{i^2 \pi^2}{L^2}\sqrt{\frac{EI}{m}} \quad , \quad a_i = \frac{i\pi}{L}$$

$$\therefore X_i(x) = A_i \sin\frac{i\pi x}{L} \qquad \text{---------- (9)}$$

Initial conditions

$$Y_i(x,o) = f_1(x) \quad , \quad \left(\frac{\partial y}{\partial x}\right)_{x,o} = f_2(x)$$

Where:

f_1 (x): the initial function of displacement of beam.

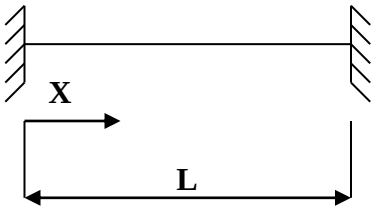
f_2 (x): the initial function of velocity of beam.

$$Y_b(x,t) = \sum_{i=1}^{\infty} Y_i = \frac{2}{L}\sum_{i=1}^{\infty} \sin\frac{i\pi x}{L}\left[\cos\omega_i t \int_o^t f_1(x)\sin\frac{i\pi x}{L}\,dx + \frac{1}{\omega_i}\sin\omega_i t \int_o^t f_2(x)\sin\frac{i\pi x}{L}\,dx\right] \qquad (10)$$

Beam With Fixed Ends

Boundary conditions

$Y_i(x,t) = o \quad and \quad \frac{\partial y_i}{\partial x} = o$ at x = o, L



Therefore:

$$\omega_i = \frac{(i+1/2)^2 \pi^2}{L^2}\sqrt{\frac{EI}{m}} \qquad \text{---------- (11)}$$

$$a_i = \frac{(i+1/2)\pi}{L}$$

$$\therefore X_i(x) = \left(\cosh a_i x - \cos a_i x - \alpha_i\left(\sinh a_i x - \sin a_i x\right)\right) \quad (12)$$

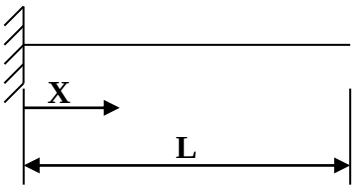
$$\alpha_i = \frac{\cosh a_i L - \cos a_i L}{\sinh a_i L - \sin a_i L}$$

Cantilever Beam

Boundary conditions

$Y_i(x,t) = o$ and $\frac{\partial y}{\partial x} = o \qquad at \qquad x = o$

$\frac{\partial^2 Y_i}{\partial x^2} = o$ and $\frac{\partial^3 Y}{\partial x^3} = o \qquad at \qquad x = L$



Therefore:

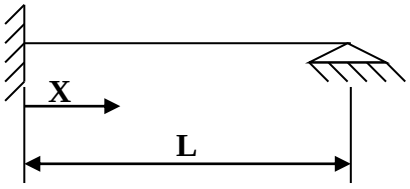
$$\omega_i = \frac{(i-1/2)^2\ \pi^2}{L^2}\sqrt{\frac{EI}{m}} \qquad \text{---------- (13)}$$

$$a_i = \frac{(i-1/2)\pi}{L}$$

$$X_i(x) = \left(\cosh a_i x - \cos a_i x - \alpha_i\left(\sinh a_i x - \sin a_i x\right)\right) \quad \text{---------- (14)}$$

$$\alpha_i = \frac{\cosh a_i L + \cos a_i L}{\sinh a_i L + \sin a_i L}$$

Fixed – Hinged Beam



Boundary conditions

$$Y_i(x,t) = o \ , \quad \frac{\partial Y_i}{\partial Y} = o \qquad at \ \ x = o$$

$$Y_i(x,t) = o \ , \quad \frac{\partial^2 y_i}{\partial x^2} = o \qquad at \ \ x = L$$

Therefore:

$$\omega_i = \frac{(i + 1/4)^2 \pi^2}{L^2} \sqrt{\frac{EI}{m}} \ , \quad a_i = \frac{(i + 1/4)\, \pi}{L} \qquad \text{----- ------- (15)}$$

$$X_i(x) = \cosh a_i x - \cos a_i x - \alpha_i (\sinh a_i x - \sin a_i x) \quad \text{--------- (16)}$$

$$\alpha_i = \frac{\cosh a_i L - \cos a_i L}{\sinh a_i L - \sin a_i L}$$

For the last three cases (fixed – fixed, cantilever, fixed-hinged) beams, the determining of the response of an elastic body to initial condition involves the evaluation of integrals of the forms

$$\int_o^L f_1(x)\ X_i(x) \quad dx \quad , \quad \int_o^L f_2(x)\ X_i(x) \quad dx$$

Direct integration to such expressions become difficult when the normal functions ($X_i(x)$) are complicated. Beams other than simply involve hyperbolic function which usually necessitate numerical integration.

However an alternative approach is found to be advantageous, especially when the initial condition is caused by a concentrated force Po (suddenly removed at time t=o) therefore the deflection of last three cases is [10]:

$$Y_i(x,t) = \frac{P_o L^3}{EI} \sum_{i=1}^{\infty} \frac{X_i \, X_{i1}}{(a_i L)^4} \cos \omega_i \, t \qquad \text{---------- (17)}$$

Where:

X_{il}: X_i evaluated at x=x1 where P_o is acting.

Forced Vibration

For the case of forced vibration, the (L.H.S.) of equation (5) is similar to that for free vibration but the external dynamic load intensity (P (x, t)) exists in the (R.H.S.) of the same equation.

Lagrange's equation may be used to solve the forced vibration equation. Let the displacement Y_b (x,t) is given by:

$$Y_b(x,t) = \sum_{i=1}^{\infty} X_i(x) . Z_i(t)$$

where X_i (x) is the characteristic shape function of the beam for ith mode which depends on the beam end conditions, Z_i(t) is the modal amplitude which can be obtained by using the Lagrange's equation [14,16,17]. For undamped vibration, the Lagrange's equation is in the following form:-

$$\frac{d}{dt}\left(\frac{\partial K.E}{\partial \dot{Z}_i}\right) - \frac{\partial K.E}{\partial Z_i} + \frac{\partial U}{\partial Z_i} = \frac{\partial Pe}{\partial Z_i} \qquad \text{---------- (18)}$$

Where:

K.E: the kinetic energy

U: the strain energy

Pe: the potential of external forces.

If (m) is the beam mass per unit length then:-

$$K.E = \frac{1}{2}\sum_{i=1}^{\infty} m\left(\dot{Z}i\right)^2 \int_o^L X_i^2(x)dx$$

$$U = \frac{1}{2}\sum_{i=1}^{\infty} EI\left(Z_i\right)^2 \int_o^L X_i''^2(x)\,dx$$

$$Pe = \int_o^L P(t,x)\left(\sum_{i=1}^{\infty} Z_i\, X_i(x)\right) dx$$

$$Pe = \int_o^L f(t) P_1(x)\left(\sum_{i=1}^{\infty} Z_i\, X_i(x)\right) dx \qquad \text{---------- (19)}$$

Where f (t) is the load-time function and P_1 (x) is the load distribution along the span (L).

Using equation (18 and 19)

$$\ddot{Z}_i + \omega_i^2 \quad Z_i = \frac{f(t)\int_o^L P_1(x) X_i(x)\,dx}{m\int_o^L X_i^2(x)\,dx} \qquad \text{---------- (20)}$$

Where ω_i, is the i^{th} mode natural frequency, and

$$\omega_i^2 = \frac{EI\int_o^L X_i''^2(x)\,dx}{m\int_o^L X_i^2(x)\,dx} \qquad \text{---------- (21)}$$

For a point load acting at distance $(\bar{x})$ from the support, the solution is given by [14]:-

$$Y_b(x,t) = \sum_{i=1}^{\infty} \frac{X_i(x)\cdot X_i(\bar{x})}{m\omega_i \int_o^L X_i^2(x)\,dx} \int_o^L F(\bar{t}) \sin\left(\omega_i (t-\bar{t})\right) d\bar{t} \quad \text{------- (22)}$$

The displacement of the beam can be determined using equation (22) and substituting in equation (4)

$$\left(\frac{F}{K}\right)^{2/3} = V_o.t - \frac{1}{m_s} \int_o^t d\tau \int_o^\tau F(\bar{\tau})\, d\bar{\tau} - \sum_{i=1}^{\infty} \frac{X_i(x).X_i(\bar{x})}{m\omega_i \int_o^L X_i^2(x)\, dx} \int_o^t F(\bar{t}) \sin\left(Wi(t-\bar{t})\right) d\bar{t} \quad \text{---- (23)}$$

This equation can not be-solved in a closed form, but it may be solved numerically to give the impact force-time history. The detailed numerical solution for simply supported beam was given in ref. (4), and is developed in this research for other types of beams.

The following quantities may then be computed.

1- The displacement (equation 22).

2- The model bending moment,

$$M_i(x,t) = -EI \frac{\partial^2 Y_i}{\partial x^2}$$

3- The model kinetic energy, KE(t)

$$= \frac{X_i^2(\bar{x})}{2m \int_o^L X_i^2(x)dx} \left(\int_o^t F(\bar{t}) \cos\left(\omega_i (t-\bar{t})\right) d\bar{t} \right)^2 \quad \text{---------- (24)}$$

4- The model strain energy Ui (t)

$$= \frac{X_i^2(x)}{2m \int_o^L X_i^2(x)dx} \left(\int_o^t F(\bar{t}) \sin\left(\omega_i (t-\bar{t})\right) d\bar{t} \right)^2 \quad \text{---------- (25)}$$

The analysis given above covers beams having any end conditions.

Simply Supported Beam

To study impact of mid span $\left(\overline{X} = L/2\right)$

$$X_i = \sqrt{\frac{2}{L}} \sin\frac{i\pi x}{L} \qquad (i = 1,3,5,.....)$$

$$\int_o^L X_i^2(x)\, dx = 1$$

The central bending moment is:

$$M\left(\frac{L}{2}, t\right) = \frac{2L\omega_1}{\pi^2} \sum_{i=1,3,5,..}^{\infty} \int_0^t F(\bar{t}) \sin\left(\omega_i (t-\bar{t})\right) d\bar{t} \quad \text{---------- (26)}$$

Where: ω_1 is the fundamental natural frequency and equal to ω_i / i^2

Beams Have Other End Conditions

a) Fixed –fixed

b) Cantilever

c) Fixed-hinge

For the three cases above

$X_i(x)$ is the same in the free vibration, and $\int_o^L X_i(x)\, dx = L$ [17]

For these three cases the bending moment M (x,t) is equal to:

$$M(x,t) = \frac{-EI}{m_b} \sum_{i=1}^{\infty} Q_b * \frac{1}{\omega_i} \int_o^t F(\bar{t}) \sin\left(\omega_i (t-\bar{t})\right) d\bar{t} \text{ ---------- (27)}$$

Where

$$Q_b = \left(2a_i^2 + 2\alpha_i a_i^2\right)\left(\sinh^2 a_i x + \cosh^2 a_i x\right)$$
$$+8\ \alpha_{i^i}\ \ a_i^2\left(\cos a_i x\ \ \sin a_i x\right) + \left(2a_i^2 - 2\alpha_i^2 a_i^2\right)$$
$$\left(\cosh a_i x\ \ \cos a_i x + \sin\ \ a_i x\ \ \sinh\ \ a_i x\right)$$
$$-\left(2a_i^2 + 2\alpha_i^2\ \ a_i^2\right)\left(\cos a_i x\ \ \cosh a_i x - \sin a_i x\ \ \sinh a_i x\right)$$
$$-4\alpha_i\ \ a_i^2\left(\sin a_i x\ \ \cosh a_i x + \cosh a_i x\ \ \sinh a_i x\right) + \left(2a_i^2 + 2\alpha_i\ \ a_i^2\right)$$
$$\left(\cos^2 a_i x - \sin^2 a_i x\right) + 4\alpha_i\ \ a_i^2$$
$$\left(\cos\ \ a_i x\ \ \sinh\ \ a_i x - \sin\ \ a_i x\ \ \cosh\ \ a_i x\right)$$

As mentioned before, the above equation may be solved numerically. A computer programme [1] is written to determine the impact force and deflection time histories and:

1- Central bending moment $\left(M\left(\frac{L}{2},t\right)\right)$

2- Kinetic energy $(K.E)_i$ (t)

3- The absorbed strain energy U_i (t).

a) To check the accuracy of the developed programme solution, Fig.(1) shows the force – time history of a simply supported beam subjected to central impact load which compared with

solution given by Eringen [15] the following parameters were used:

ms: Striker mass = .03334 kg.

mb: Beam mass = 0.1222 kg.

Vo: Impact velocity of striker = 1 m/sec

K: Deformation constant = 13.75E 6 kg/m $^{1..5}$

ω_1: 531 rad/sec

b) For the present search the input data in the computer programme are:[1]

ms: Striker mass = 10 kg.

mb: Beam mass = 20 kg.

Vo: Impact velocity of striker = 2 m/sec

K: Deformation constant = 24.7 x 10^6 N/m $^{1..5}$

N: Number of excited modes = 6

L: length of Beam = 0.5 m

E: Yong modulus of elasticity = 25.6 x 10^9 N/m^2

I: Moment of inertia = 3.90625 x 10^{-7} m^4

CONCLUSIONS

The theoretical results obtained in the present analysis are compared with experimental and theoretical works previously done. A good agreement is found between theoretical and experimental results Fig. (1). this indicates that the impact resistance of relatively large beams may be predicted by using the theoretical approach based on equation of undamped beam

vibration. All the derivations required to predict the effects of boundary conditions are performed for both forced and free vibrations. The impact force and deflection-time histories Figures (2), (3), (4) and (5). The strain energy absorbed by the beam Figure (9) and the bending moment Figure (8) are all determined.

Figures (6) and (7) show that for the same falling mass and the same applied kinetic energy (height of drop) for all cases, the maximum central deflection and the maximum impact force are affected by the boundary conditions of the beams. The maximum central impact force for fixed –fixed beam is higher than other cases by about (27%) for simply supported beam, (16%) for fixed-hinge and (4%) for square cantilever beam, the maximum central deflection for cantilever beam is smaller than other cases by about (67%) for simply supported beam, (38%) for fixed-hinge beam and (24%) for fixed –fixed beam.

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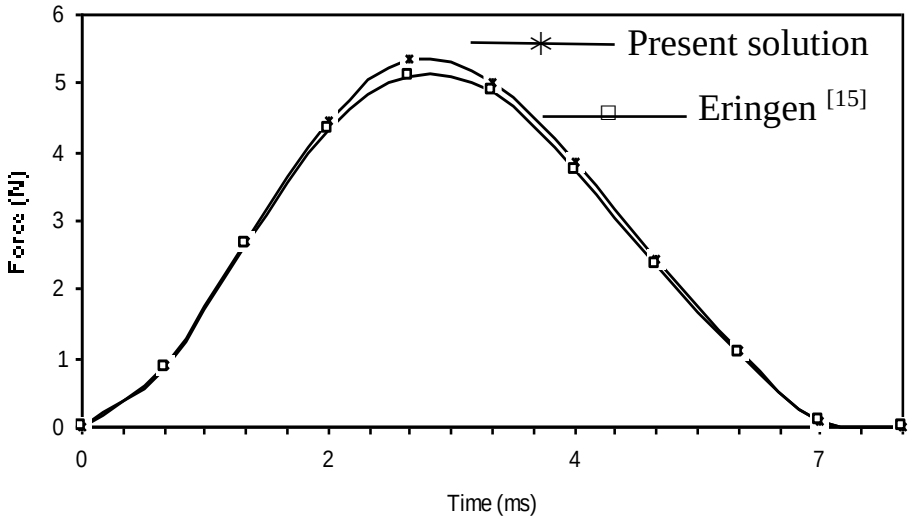


Fig. (2) Typical Theoretical Impact Force – Time History for Simply Supported Beam.

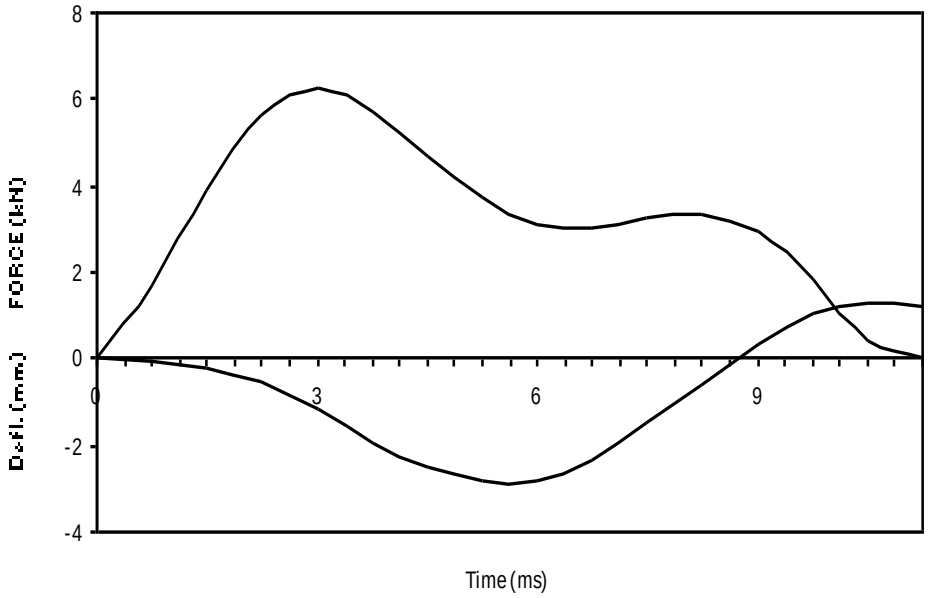


Fig. (3) Typical Theoretical Impact Deflection and Force – Time ,Histories for Simply Supported Beam.

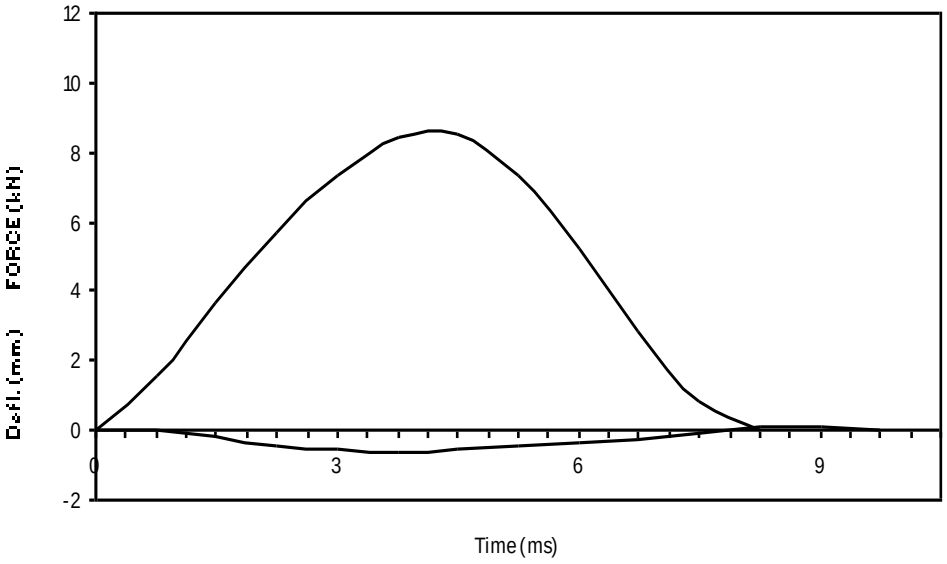


Fig. (3) Typical Theoretical Impact Deflection and Force – Time Histories for Fixed - Fixed Beam.

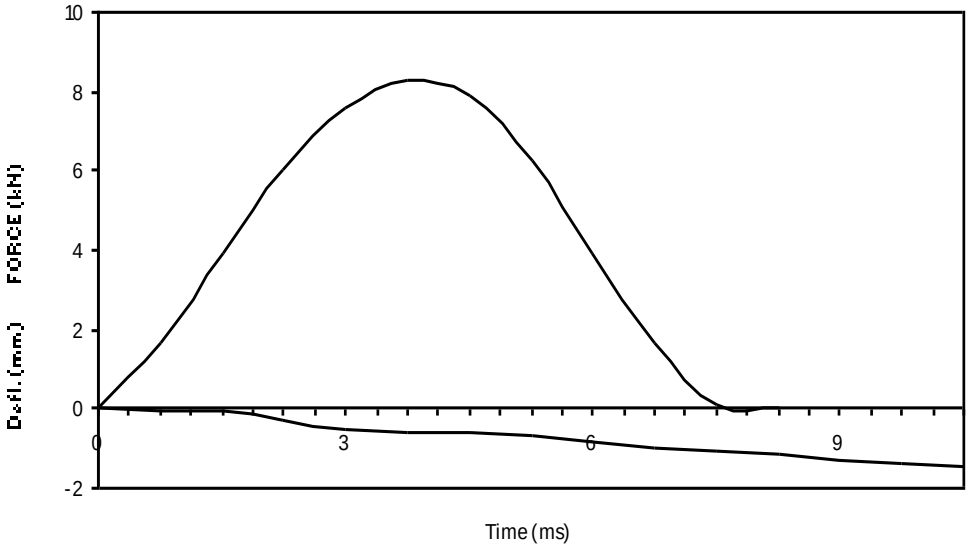


Fig. (4) Typical Theoretical Impact Deflection and Force – Time Histories for Fixed - Free Beam.

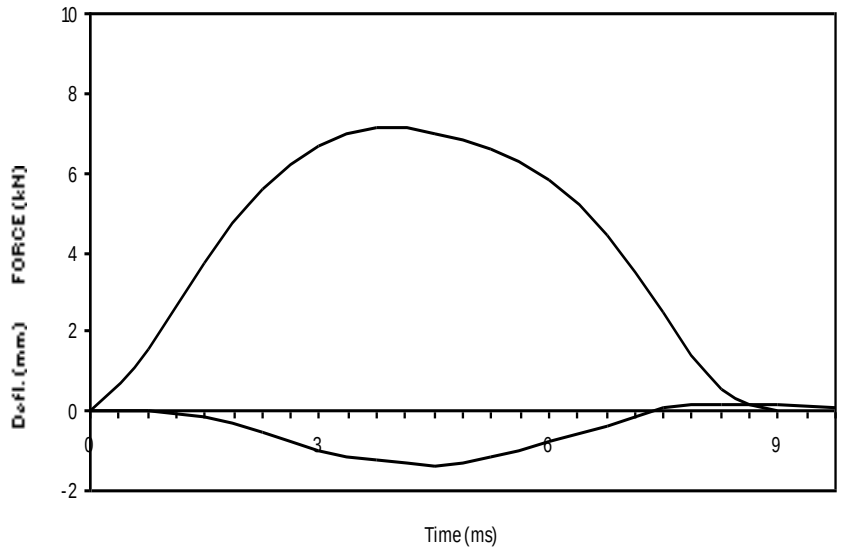


Fig. (5) Typical Theoretical Impact Deflection and Force – Time ,Histories for Fixed - Hinge Beam.

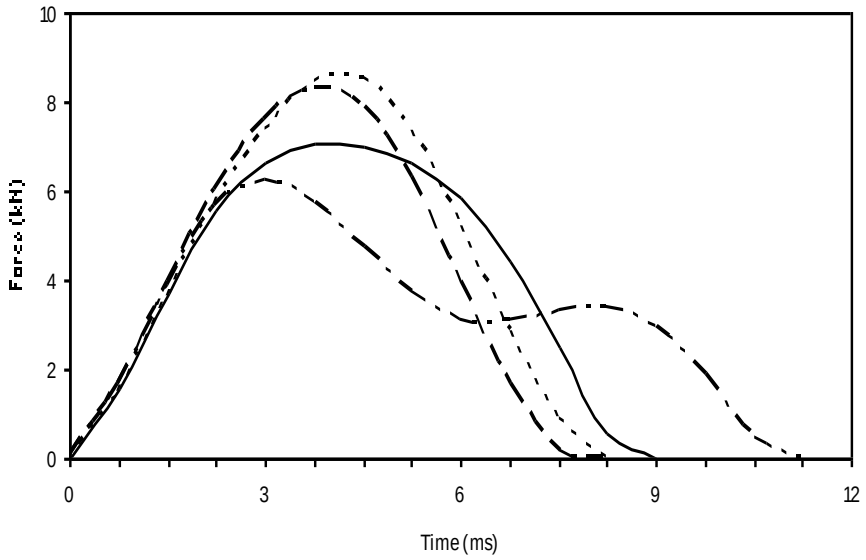
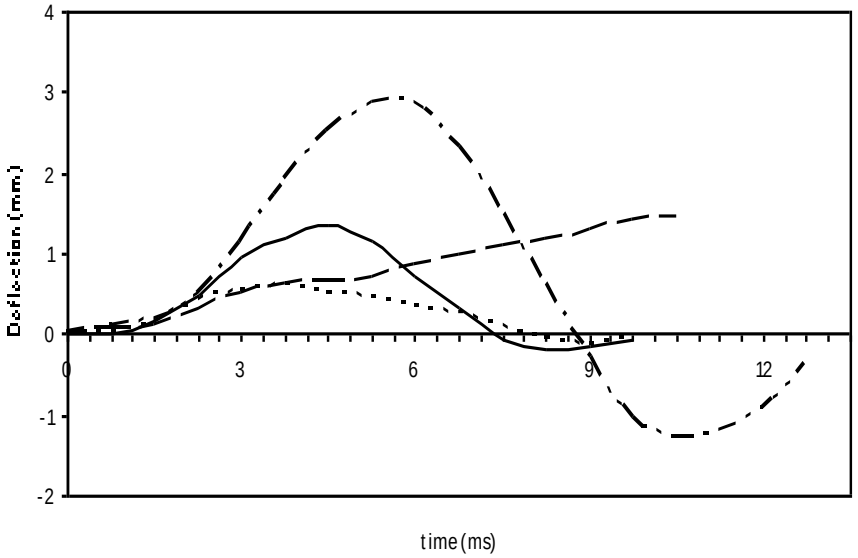


Fig. (6) Typical Theoretical Impact Force – Time History for Different End Condition.



Simply Supported Beam.
Fixed – Fixed Beam.
Cantilever Beam.
Fixed – Hinged Beam.

Fig. (7) Typical Theoretical Impact Deflection– Time History for Different End Condition.

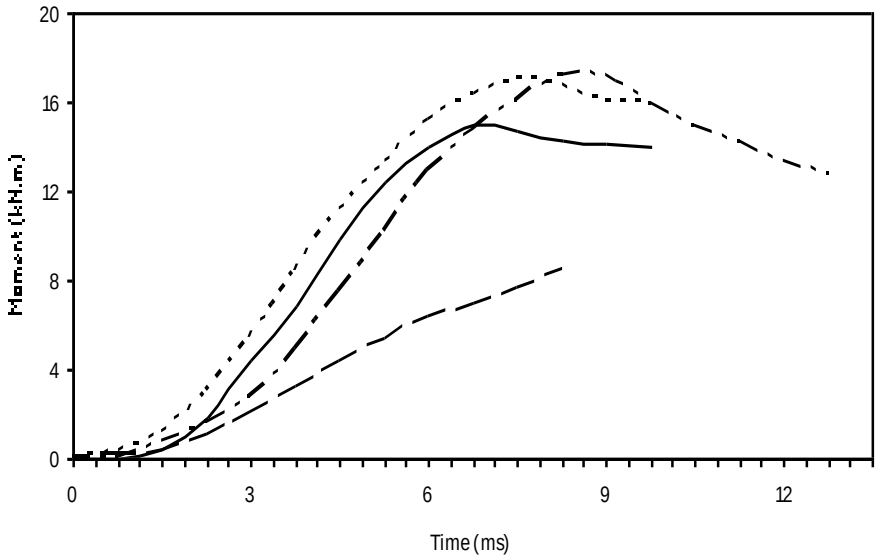
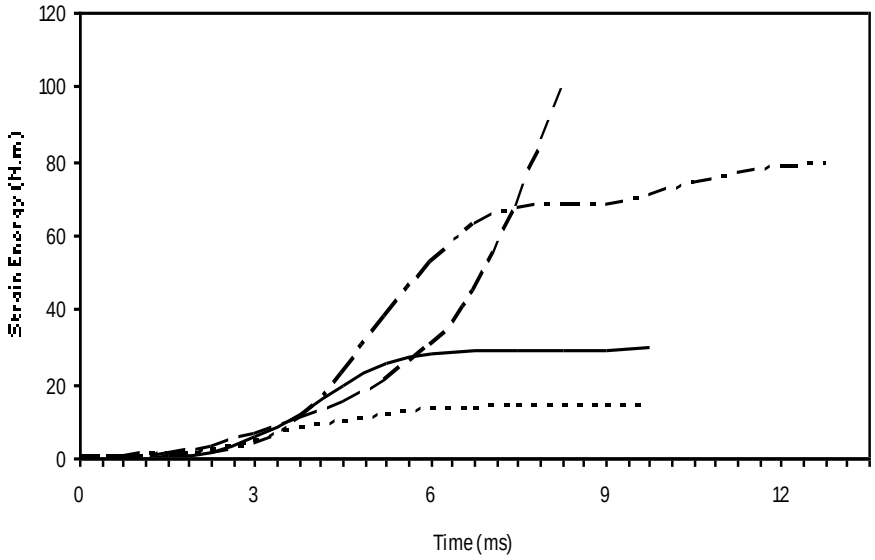


Fig. (8) Typical Theoretical Impact Moment– Time History for Different End Condition.



—·—·—·— Simply Supported Beam.
····· Fixed – Fixed Beam.
- - - - - - - - Cantilever Beam.
———— Fixed – Hinged Beam.

Fig. (9) Typical Theoretical Impact Strain Energy – Time History for Different End Condition.

تأثير الشروط الحدية على اجهادات الصدم في العتبات

د. اياد كاظم صيهود الحجامي
قسم هندسة البناء والانشاءات–الجامعة التكنولوجية

الخلاصة

يشمل هذا البحث دراسة نظرية حول تأثيرالشروط الحدية على العتبات تحت تأثير الاحمال الصدمية .لقد تم خلال هذا البحث اجراء تحليلات نظرية لتأثير احمال الصدم على العتبات لمختلف الشروط الحدية وذالك بحل معادلة الصدم التكاملية للعتب بالطرق العددية وبأستعمال الحاسب الالي, كما تم حساب الدالة الزمنية للقوة المسلطة على العتبة والأود المركزي الناتج وطاقة الأجهاد الممتصة من قبل العتب وكذالك مقدار العزوم خلال فترة الصدم مع الاخذ بنظرالاعتبارالاهتزاز المقيد والحر. اظهرت الدراسة تقارباً بين نتائج المستحصلة عند مقارنتها مع النتائج العملية السابقة . كما اظهرت النتائج لنفس الكتلةِ السَاقِطةِ ونفس الطاقةِ الحركيّةِ (إرتفاع الهبوطِ) لكُلّ الحالات التي تمت دراستها بأن القوة الصدمية والهطول عند منتصف العتبة يتأثر بالشروط الحدية للعتبات.