Development of New Algorithm for Communication Networks Reliability Based on Tie Set Method Combined with a Modified Flooding Algorithm

Dr. Musaria Karim Mahmood Department of Electrical Engineering, Tikrit University

Abstract

This paper presents a new method to calculate the network reliability based on the use of flooding routing algorithm. Tie Set (TS) method is one of probabilistic reliability method, is combined with flooding technique to generate an efficient new method to estimate reliability. The proposed method can be generalized to be active with others reliability methods such as Cut Set, and Enumeration methods.

TS method depends on two factors: finding the TS paths and the inclusion–exclusion expansion equations. A modified flooding algorithm is used to generate the group of TS paths which is then used to find the reliability. It will be demonstrated by a case experiment the simplicity and effectiveness of the flooding technique to generate paths between a pair of nodes in a graphical representation of a communication network.

Keyword: Communication Network, Reliability, Tie Set, Flooding Algorithm, Graph.

يقدم هذا البحث طريقة جديدة لحساب موثوقية شبكة اتصالات تعتمد على استخدام خوارزمية التوجيه باغراق الشبكة. طريقة تحديد مجموعة الوصلات (Tie Set) وهي واحدة من طرق احتساب الموثوقية بالطرق الاحتمالية ، مستخدمة هنا جنبا إلى جنب مع تقنيات الفيضانات لتوليد طريقة جديدة فعالة لتقدير الموثوقية. الطريقة المقترحة يمكن أن تكون معممة لتستخدم مع الطرق الاحتمالية الاخرى مثل طريقة تحديد مجموعة القطوع (Cut) وطريقة التعداد التفصيلي (Enumeration) .

طريقة تحديد مجموعة الوصلات تعتمد على عاملين: العثور على كافة المسارات TS وتنفيذ المعادلات الرياضية الخاصة باحتساب الموثوقية. تستخدم خوارزمية الفيضانات المعدلة لتوليد مجموعة من المسارات TS الذي يستخدم بعد ذلك لاحتساب الموثوقية. سيظهر ذلك من خلال تنفيذ الخوارزمية على شبكة اخذت كمثال للتاكد من فعالية الطريقة المبتكرة في هذا البحث.

Introduction

Reliability is the probability that the system operates successfully for a given period of time under environmental conditions. Network topology reliability is a key point of the present network reliability researches, which is the study of network reliability using graph theory from the aspect of network topology structure ^[1].

In managing a communication network and planning topological modifications it is important to be able to determine reliability measures quickly. In the most general case, we would like to determine a precise relationship between the failure of network components and the amount of traffic the network can handle. Such analysis is usually quite complex and time consuming as it involves not only a combinatorial analysis of the states arising from failed components, but also an analysis of the routing within the network ^[2].

The layout of the paper is as follows: The second section presents the literature review of previous works related to reliability calculation. In section three, the Tie Set method is introduced, with main equation used to calculate reliability. In section four, the principal of flooding routing algorithm is presented. The proposed algorithm is presented in section five with the mathematical modeling, a case study is examined and results are presented. The final section concludes the paper.

Literature Review

The study of network reliability has led to a huge body of literature. The calculation of network reliability in a probabilistic context has long been an issue of practical and academic importance. Conventional approaches such as determination of bounds sums of disjoint products algorithms, Monte Carlo evaluations, studies of the reliability polynomials, etc.. only approximations provide when the network's size increases^{[3] [4]}

A. M. Shooman in 1991^[5] investigated the calculation of exact reliability of communication network using graph reduction technique. M. L. Shooman in 2003^[6] analyzed the main methods for calculating reliability based on probabilistic graph theory. This work can be considered as survey of methods. Older method, the states enumeration method is presented in two forms, the exact and the approximated. It had found

that this method is applicable for only small networks. S. Soh, and S. $Rai^{[7]}$ proposed the use of Tie/ Cut Set method to evaluate communication network reliability with heterogeneous link capacities. They conclude that this method can be used efficiently for large communication network. C. Tanguy in 2007 [8] presented an exact method for the calculation of two-terminal reliability for directed network. S. Kuo, et al. in 2007 ^[9] defined a more general concept of kterminal reliability, where k - terminals must be connected. If k=2, we have twoterminal reliability, while in the case k=all terminals, we have all-terminal reliability. Thus k-terminal reliability can be viewed as a more general concept. Hou et al. in 2010^[1] presented a Markov-based algorithm for communication network topology reliability. and tries to prove its rationality.

Other works exist to evaluate reliability of electronic systems, control systems, pipeline and refinery in petroleum industry and power production plants based on the same theory used to evaluate communication network reliability using graph theory.

A. G. Bruce in 1998 ^[10] proposed the analysis of Supervisory Control and Data Acquisition (SCADA) system reliability in terms of its expected, aggregate contribution to load curtailment on the power system. Li and Zhao in 2005^[11] present a cut/Tie Set method to evaluate reliability of control systems bv searching for the equivalent TS based on the control system performance. When a fault is detected and identified on-line and/or the control objective is changed, the reliability can be easily re-evaluated by updating the TS. Siddiq in 2006^[12] studied a SCADA system for the Iraqi-Turkish pipeline and evaluate the reliability of the network used as

backbone of data exchange using TS method. M. K. Mahmood in 2012 ^[13] uses a Tie Set method to evaluate a new designed Intranet for refinery industry in Iraq.

Tie Set Reliability Calculation Method Reliability calculation is mainly based on mathematical development of graph theory and probability theory. The simplest way to understand reliability calculation is to begin by introducing two-terminals reliability. A pair of nodes is used as basic of calculation, the source node (n_s) and the destination one (n_d) . The application of one of reliability methods yields to finding the reliability between this pair of node (R_{sd}) .

The all-terminal reliability (this is sometimes termed overall network reliability) problem is somewhat more difficult than the two-terminal reliability problem. Essentially, we must modify the two-terminal problem to account for all-terminal pairs. All-terminal reliability is the probability that a set of operational edges provides communication paths between every pair of nodes, while in the case of two-terminal problem one pair (n_s, n_d) is considered ^[6].

The reliability methods using probabilistic approach are:

- State Space Enumeration method (SSE)
- Graph Transformation method (GT)
- Tie-Sets (TS) and Cut-Sets (CS)
- Approximated methods.

In this work we are interested in Tie Set method, which is a very important method used to find the reliability for medium to large communication network both for two or all-terminals models. If the number of network links is equal to (e) then the reliability computing complexity of SSE method is of order 2^e, while GT method complexity depends on the reduction technique used to simplifying the graph representing the communication network. The reliability computing complexity can be reduced below the 2^e required for the SSE method by the use of minimal cut sets (CS) and minimal tie sets (TS) methods^[6].

TS are the groups of edges that form a path between nodes (n_s) and (n_d) . The term minimal implies that no node or edge is traversed more than once (loop free). Similarly, one can focus on the minimal CS of a graph. CS is a group of edges that break all paths between (n_s) and (n_d) when they are removed from the graph. If CS is minimal, no subset is also a CS.

If there are (i) TS between (n_s) and (n_d) , then the reliability expression is given by the expansion of ^[6]:

$$R_{sd} = P(T_1 + T_2 + \dots + T_i)$$
(1)

Where R_{sd} represents the two terminal reliability between node (n_s) and node (n_d) which means the probability that node (n_s) and (n_d) are connected, T_m is the TS number (m), P is the probability of union of all TS.

If TS $(T_1, T_2, ..., T_i)$ are all disjoints (or mutually exclusive) then (1) can be written as:

$$R_{sd} = P(T_1) + P(T_2) + \dots + P(T_i) \quad (2)$$

But usually different TS (paths) are not disjoints. TS is composed by links to connect the source node (n_s) to the destination (n_d) , then two different TS (paths) can have same links in part.

$$R_{sd} = P(T_1 + T_2) = P(T_1) + P(T_2) - P(T_1T_2)$$
(3)

Suppose that all links have same properties then the probability of each link is (p), then:

$$P(T_1) = P(L_1L_2L_7) = p \times p \times p = p^3$$

$$P(T_2) = P(L_5L_6L_7) = p \times p \times p = p^3$$

$$P(T_1T_2) = P(L_1L_2 L_5L_6L_7)$$

$$= p \times p \times p \times p \times p = p^5$$

The application of (3) yields to:

$$R_{sd} = P(T_1) + P(T_2) - P(T_1T_2) = p^3 + p^3 - p^5 = 2p^3 - p^5$$

So equation (2) cannot be used to develop the reliability. For the general case the reliability is found by the following equation:

$$\begin{split} R_{sd} &= P(T_1 + T_2 + \dots + T_i) = \\ [P(T_1) + P(T_2) + \dots + P(T_i)] - \\ [P(T_1T_2) + P(T_1T_3) + \dots + \\ P(T_rT_k)_{r \neq k}] + [P(T_1T_2T_3) + \\ P(T_1T_2T_4) + \dots + P(T_rT_kT_j)_{r \neq k \neq j} + \\ \dots + (-1)^{i-1} [P(T_1T_2T_3 \dots T_i)]. \end{split}$$

$$(4)$$

The equation (4) is the *inclusion exclusion expansion function* used to evaluate the reliability for TS and CS methods. The complexity of TS methods depends on two factors: the order of complexity involved in finding the TS and the order of complexity for the inclusion–exclusion expansion^[7]. In the present paper we focus onto the

development of new algorithm to find the set of TS using flooding technique. Equation (4) can be implemented easily using new programming techniques and must not represent a big deal for a professional programmer.

Flooding Routing Algorithm

Routing algorithm is implemented in communication network nodes, tries to construct routing tables composed of decision making to route packets or data into the network from the source to the destination. There are many routing algorithm types, trying all to find the best route between a pair of nodes.

Flooding algorithm is one of them known by its simplicity, robustness and high reliability ^[14]. Flooding is a routing algorithm in which every incoming packet is sent through every outgoing link. Flooding (also called broadcast routing) sends distinct packet to every host except the line on which it arrived, so it generates a large number of packets as in the figure 1.

This algorithm is very simple to implement but be costly in terms of wasted bandwidth, used for reliable and critical applications where the bandwidth not representing big deal such as military applications for example in Command, Control, Communication and Intelligent systems (C³I systems).

New researches especially in the field of wireless sensor networks and ad-hoc network were recently published based on the use of flooding algorithms. Ad-hoc network doesn't need fixed infrastructure and has a strong ability of anti-destroying, so it adapts on-demand routing protocols which mainly utilize the flooding mechanism^[15], ^[16].

The Proposed Method *Problem Modeling*

Many physical problems as communication network, computer networks, piping systems, and power grids can be modeled by a graphical model. In the context of this work, a graph is representing a connected communication network with (N) nodes, and (L) Links.

The graph G = (N, L), where N is a set of nodes (also called vertices) and L is a set of directed links (or edges). Each node $n \in N$, has a probability equals to (p_n) , and each link, $l \in L$, has a probability equals to (p_1) to operate correctly. Failures of the different constituents are assumed to occur at random, and to be statistically independent events. By the use of material redundancy, nodes can be be perfect with assumed to the $p_n = 1 (100\%).$ probability This assumption will simplify the development of the algorithm, but a general solution will be suggested in next sections for how to resolve the problem if nodes are not perfects.

The graphical representation of communication network is accomplished via matrix notation. We define the probability matrix (*PM*) composed by elements (l_{kj}) represents link between node (n_k) and node (n_j) . Links can have three possible ranges of values:

$$l_{kj} \begin{cases} 0 \text{ if no link between } (k) \text{ and } (j) \\ p_{kj} \text{ if link between } (k) \text{ and } (j) \\ 1 \text{ for the diagonal elements} \end{cases}$$
(5)

PM will be a square matrix with dimensions $(N \times N)$, and give a complete description of the communication network topology. For example if we consider that all links are bidirectional with same probability (p),

the matrix representing the graph of the network of figure1 is given by:

	1٦	р	0	0	0	0	p_{T}	
	p	1	p	0	0	0	p	
	0	р	1	р	0	0	0	
PM =	0	0	р	1	р	р	0	(6)
	0	0	0	р	1	0	p	
	0	0	0	р	0	1	p	
<i>PM</i> =	p	p	0	0	p	p	1	

The nodes matrix (NM) representing a row matrix of dimension $(1 \times N)$ is defined, and will be useful to generate the group of all TS. *NM* represents the packet circulating into the network from the source node (n_s) to the destination node (n_d) . Element NM_k represents the node (n_k) of the network, and if the packet pass through the node (n_k) then NM_k is set to 1, otherwise the value of NM_k is set to 0. Elements of NM are all set to 0 at the beginning, except values of NM_s is set to 1.

 $NM = \begin{bmatrix} NM_1 & NM_2 & NM_3 & \dots & NM_N \end{bmatrix}$ (7)

Finally the routes matrix RM is defined to represent the path of each TS. Each element represents a link in the network, so the dimension of this matrix will be $(N \times N)$. In the beginning all elements are initialized to 0, and by passing packets in the link RM_{kj} (link between node n_k and node n_j) will set the value of element RM_{kj} to 1. This matrix will be very useful to find all TS by translating the matrix NM to RM and then finding TS.

$$RM = \begin{bmatrix} RM_{11} & \cdots & RM_{1N} \\ \vdots & \ddots & \vdots \\ RM_{N1} & \cdots & RM_{NN} \end{bmatrix}$$
(8)

Algorithm Based on Flooding Technique

The routing problem is considered as: Known the source node (n_s) and the destination node (n_d) , look for a route between (n_s, n_d) which makes the routing cost of data packets as minimum as possible. Flooding has different approach that each packet will be forwarded to next nodes until the destination is reached, so that, many routes are generated.

The developed algorithm uses the property of flooding (forwarding packets) to implement a procedure to find all TS for a graph. The algorithm is based on three steps as shown in figure 2 below: the initialization, flooding technique, and TS reliability method.

Initialization

(A) Enter the network topology by entering the matrix PM which describes the connectivity of the network. PM, defined in (6), assigns three possible values to each possible link to form a $N \times N$ matrix. Active links are thus with probability equal to a value ($1 > p_i > 0$), while the value 0 is reserved when no link, and 1, for the diagonal of the matrix which describe the perfect node.

If nodes are imperfect then a solution is possible by a considering each node (n) as two nodes with link between them having probability equal to the given probability of the imperfect node.

(B) Initialization of the matrix *NM*, which is defined in (7), by setting all elements to 0, where each element represents a well defined node. Set element which correspond to the source node to 1.

(C) Setting all elements of matrix *RM* to a value equal to 0.

Flooding procedure

(i) After initialization of all matrices, NM is injected into the network from the source node (n_s) . This is accomplished by reading the connectivity of the network from PM, and then forwarding a copy of NM to next nodes. The location of the new node in the matrix NM is then set to 1. This process (flooding the network with NM) will continue until the destination is reached.

(ii) The flooding procedure developed in this paper is modified to be *loop free* algorithm because the original algorithm can have loops. This is to avoid the possibility of *dead look* situation presented in figure 3 where *NM* looping yields to incorrect situation. The modification can be accomplished via a test in the program if the location of new arriving node (n_k) is equal to 1 in *NM*, that is mean that this node is "*déjà vu*", and the route is cancelled.

(iii) Arriving to the destination node (n_d) , the matrix *NM* represents all nodes through which the packet pass from (n_s) to (n_d) . These nodes have their indices (corresponding locations in *NM*) equal to 1.

(iv) A problem arises that two routes can have the same matrix *NM*, but differ by the order of passage. The two routes must taking in account, as will be mentioned in the case study. An original procedure is developed to resolve this problem by introducing *binary code of order of passage*. The code will assign to each passage of new node a binary code to identify the exact path for example, for a 10, nodes network, if:

Route 1 passing through nodes: $3 \rightarrow 6 \rightarrow 8 \rightarrow 2 \rightarrow 5$

Matrix NM_1 (of route 1) will be: $NM_1 = [0 \ 1 \ 1 \ 0 \ 1 \ 0 \ 0]$ Route 2 passing through nodes: $3 \rightarrow 8 \rightarrow 6 \rightarrow 2 \rightarrow 5$

Matrix NM_2 (of route 2) will be: $NM_2 = [0 \ 1 \ 1 \ 0 \ 1 \ 1 \ 0 \ 0]$

 $NM_1 = NM_2$, but they are two different routes.

The binary code will begin by setting the order or "code" of source node (n_s) to (1-binary), and continue by increasing order to assign a code for each node. The number of bits used for this code is (b):

$$2^b - 1 \ge N \tag{9}$$

N: Network nodes number.

This code is used to construct the matrix *RM* which represents the link connection of each path, then the TS. Each TS will be represented by a different *RM* matrix.

These matrices are used in the inclusion expansion TS equation defined in (4) used to calculate the reliability.

<u>TS method</u>

The application of the TS will be forward. This procedure can be used to estimate the reliability using CS method or enumeration methods by changing only this step.

Case Study Reliability Finding

To validate the developed algorithm, the reliability of the network given in figure 4 is evaluated between node (1) and node (5). The network if formed by (8) nodes and (26) links. Figure 5 represents the flooding process to find possible paths between (1) and (5) representing TS of this network.

Flooding packet or matrix *NM* is forwarded at each node and after a maximum of six hopes, all TS are found. There are 13 possible TS listed below:

 $T_1 = L_1 \ L_{10} \ L_{12},$ $T_2 = L_1 \ L_{10} \ L_{24} \ L_3$

$$\begin{array}{l} T_{3} = \ L_{1} \ L_{2} \ L_{3} \ , \\ T_{4} = \ L_{1} \ L_{2} \ L_{11} \ L_{12} \\ T_{5} = \ L_{1} \ L_{2} \ L_{11} \ L_{13} \ L_{18} \ L_{17} , \\ T_{6} = \ L_{8} \ L_{12} \\ T_{7} = \ L_{9} \ , \\ T_{8} = \ L_{8} \ L_{24} \ L_{3} \\ T_{9} = \ L_{8} \ L_{13} \ L_{18} \ L_{17} , \\ T_{10} = \ L_{8} \ L_{23} \ L_{2} \ L_{3} \\ T_{11} = \ L_{7} \ L_{19} \ L_{18} \ L_{17} , \\ T_{12} = \ L_{7} \ L_{19} \ L_{26} \ L_{24} \ L_{3} \\ T_{13} = \ L_{7} \ L_{19} \ L_{26} \ L_{12} , \end{array}$$

As presented in figure 5, two paths can have same matrix *NM*, so to avoid confusion *NM* is translated to *RM*, which gives directly the TS. For example T_2 , T_4 , and T_{10} , have all the same matrix *NM* because they pass exactly by the same nodes (1,2,3,4, and 5) as in the matrix:

 $NM = [0 \ 0 \ 0 \ 1 \ 1 \ 1 \ 1 \ 1]$

But the corresponding *RM* matrices differ because they pass through different links, from equation (8) *RM* matrix is:

	<mark>0</mark> ٦	1	0	0	0	0	0	0	
	0	0	0	1	0	0	0	0	
	0 0 0	0	0	0	1	0	0	0	
$RM(2) = T_2 =$	0	0	1	0	0	0	0	0 0	
$RM(2) = I_2 =$	0	0	0	0	0	0	0	0	
	0	0	0	0	0	0	0	0 0	
	00	0	0	0	0	0	0	0	
	L _O	0	0	0	0	0	0	01	
	۲ 0	1	0	0	0	0	0	ן0	
	0	0	1	0	0	0	0	0	
	0 0	0	0	1	0	0	0	0	
$RM(4) = T_4 =$	0	0	0	0	1	0	0	0 0 0 0	
$I(1)(1) = 1_4 =$	0	0	0	0	0	0	0	0	
	0	0	0	0	0	0	0	0	
	0000	0	0	0	0	0	0	0	
	Lo	0	0	0	0	0	0	പ	

	L0	0	0	1	0	0	0	ך0
	0	0	1	0	0	0	0	0
	0	0	0	0	1	0	0	0
$RM(10) = T_{10}$	= 0	1	0	0	0	0	0	0
$I_{10}(10) = I_{10}$	- 0	0	0	0	0 0	0	0	0
	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0
	LO	0	0	0	0	0	0	01

RM matrix will be used twice, one time is used to find the TS group, and second time in the implementation of equation (4) the exclusion expansion function.

Figure 6 shows how binary code is used to generate node passage order, which is used to translate *NM* matrices into *RM* matrices for *RM*(2) corresponding to T_2 , and *RM*(10) corresponding to T_{10} . In this case study there are 8 nodes, so 4-bits are enough to generate this order because from equation (9):

 $2^4 - 1 = 15 \ge N \ (8 \ nodes)$

After finding all TS of this graph the reliability calculation is a direct application of equation (4).

Conclusions

Tie Set method can be used for medium to large networks size requiring less computation than the enumeration method. Tie-Set is very efficient but there is some difficulties in finding all TS group (paths). The developed algorithm is based on the simple theory of flooding routing algorithm. This method is proven by the case study to be active and easy to implement. The implementation of the algorithm can be accomplished by any programming technique.

This algorithm can be generalized for Cut Set method and enumeration method by changing the inclusion equation only. The modification can be made to count CS instead of TS, which have the same approach.

Applying this algorithm for all-terminal reliability will be a simple repetition of

the algorithm used in two-terminal for all possible source-destination pair nodes.

References

- 1. D. Hou, N. Huang, and Y. Chen, "An Evaluation Method for Communication Network Topology Reliability Based on Markov Model", 2010 IEEE International Conf. Educational on and Technology (ICEIT Information 2010), Vol. 2, pp. 345- 349.
- A. M. Shooman, A. M., and A. Kershenbau, "Methods for Communication-Network Reliability Analysis: Probabilistic Graph Reduction", Proceed. Annual Reliability and Maint. Symp. 1992. IEEE, NY, pp. 441–448.
- M. Daoud, and Q. H. Mahmoud, "Estimating the Task Route Reliability of Mobile Agent- based Systems using Monte Carlo Simulation", Proceed. of the 10th IEEE Symp. on Computers and Comm. (2005). pp. 569-574.
- 4. M. Chari, and C. J. Colbourn, "Reliability polynomial: a survey", Journal of combinatorial information and system science, Vol. 22, pp. 177-193, (1997).
- A. M. Shooman, "Exact Graph -Reduction Algorithms for Network Reliability Analysis" Global Telecommunications Conf., AZ, USA Dec 1991, pp. 1412-1420.
- 6. M. L. Shooman , 'Reliability of computer systems and networks: fault tolerance, analysis, and design', John Wiley & Sons, Inc. 2002.
- Sieteng Soh, and Suresh Rai," An efficient Cut- set approach for evaluating Communication network reliability with heterogeneous linkcapacities", IEEE Trans. On

reliabilityVol. 54, No.1, March 2005. pp.133-144.

- C. Tanguy, "Exact two-terminal reliability of some directed Networks", 6th International Workshop on Design and Reliable Comm. Networks, DRCN. France Oct. 2007, pp.1-8.
- 9. S. Y. Kuo, F.Yeh, and H. Y. Lin, "Efficient and exact reliability evaluation for networks with imperfect vertices. IEEE Trans. On reliability, Vol. 56, No. 2, June 2007. pp.288-300.
- A. G. Bruce, "Reliability analysis of electric utility SCADA systems", IEEE Trans. On Power Systems, Vol. 13, No. 3, August 1998, pp.844-849.
- H. Li, and Q. Zhao, "A Cut/Tie Set Method For Reliability Evaluation of Control systems", 2005 American Control Conf. Portland.
- H. I. Siddiq, Computer-intelligent instrumentation system prototypes for oil industry applications, PhD Thesis, Al- Nahrain University, Iraq 2006.
- Musaria K. Mahmood, Fawzi M. Al-Naima, and N. K. Uzunoglu," Design and simulation of a data transmission network for industrial control system subject to reliability improvement", 2012 Intern. Conf. on Future Communication Networks (ICFCN'12), Baghdad.
- 14. L. Guo, Y. Peng, X. Wang, D. Jiang, and Y. Yu, "Performance evaluation for on-demand routing protocols based on OPNET modules in wireless mesh networks", Computers and Electrical Engineering, Elsevier, Vol. 37 (2011), pp.106–114.
- 15. W. Chen, N. Rao, D. Liang, and W. Huang, "An Ad Hoc Routing Algorithm of Low- Delay Based on Hybrid Particle Swarm

Optimization" IEEE, ICCCAS 2008, pp. 394-397

16. J. Jun, M. L. Sichitiu, "MRP: Wireless mesh networks routing protocol", Computer Comm., Elsevier, Vol. 31 (2008) pp.13–35.



Figure (1): Flooding process



Figure 2 : Developed algorithm



Figure (3): Dead lock situation



Figure (4): Case study 8-nodes network



Figure (6): Binary code used to identify order of passage