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Design of Fractional–PID Controller Based on Grey Wolf Optimization for Robotic Manipulator

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Keywords:
PUMA 560 robot; Fractional Order PID controller; Classical PID controller; Grey wolf Optimization.

Highlights:

- *GWO-optimized FOPID controller designed for PUMA 560 robotic manipulator.*
- *Superior tracking: Near-zero overshoot & faster settling vs PID/others.*
- *Robust under max load: Maintains performance where classical PID degrades.*
- *GWO tuned FOPID gains & fractional orders (λ, μ) minimizing ITAE index.*

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Abstract: To overcome the tracking issue with robot manipulators, a modern control strategy is suggested. In this method, the system, controlled by a Fractional-order PID controller strategy, is initially modeled using information that is only partially known. A wolf grey optimization has been used to find the optimum time response, fine-tuning for variable parameter gains, which are then added to the resultant controller to approximate the ignored dynamics and modeling flaws. The suggested method is systematic and draws on principles of stabilizing joint control from well-known nonlinear dynamics. In the final step, the simulation results were acceptable and compared with several control strategies. The results obtained a superior response with a minimum execution time compared to others. FOPID based on GWO is introduced to derive the adaptation laws for variable good parameters. Computer simulation using the PUMAROBOT 560 (PR560) has been employed to demonstrate the effectiveness of the proposed controller.

تصميم متحكم PID كسري الترتيب باستخدام تحسين الذئب الرمادي لتطبيقات الروبوتات المناولة

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الخلاصة
للتعامل مع مشكلة التتبع في الروبوتات المناولة، تم اقتراح استراتيجية تحكم حديثة. في هذه الطريقة، يتم أولاً نمذجة النظام -الذي يتم التحكم فيه باستخدام إستراتيجية متحكم PID كسري الترتيب- باستخدام معلومات معروفة جزئياً فقط. تم استخدام تحسين الذئب الرمادي (GWO) للحصول على أفضل استجابة زمنية، مع ضبط دقيق لمكاسب المعاملات المتغيرة، والتي تُضاف لاحقاً إلى المتحكم الناتج لتقريب الديناميكيات المُهملة وأخطاء النمذجة. تستند الطريقة المقترحة إلى منهجية نظامية وتعتمد على مبادئ تحكم المفاصل الثابتة المستمدة من الديناميكيات اللاخطية المعروفة. في الخطوة الأخيرة، كانت نتائج المحاكاة مقبولة وتمت مقارنتها مع عدة استراتيجيات تحكم أخرى، حيث حققت النتائج استجابة متفوقة بأقل وقت تنفيذ مقارنة بالأساليب الأخرى. تم تقديم متحكم PID كسري الترتيب المعتمد على تحسين الذئب الرمادي (FOPID-GWO) لاشتقاق قوانين التكيف للمعاملات الجيدة المتغيرة. واستُخدمت محاكاة حاسوبية بواسطة الروبوت الصناعي (PR560) PUMA 560 لإثبات فعالية المتحكم المقترح.

الكلمات الدالة: روبوت PUMA 560، متحكم PID كسري الترتيب، متحكم PID التقليدي، تحسين الذئب الرمادي.

1.INTRODUCTION

Control theory for nonlinear systems has been the subject of numerous articles and studies. Most of the results initially required modern control and full-state feedback. To efficiently regulate a nonlinear electrically driven robotic system, a resilient fractional-order proportional-derivative control structure, as well as the structure of a control fractional-order integral (FOFPD + FOI), were introduced in [1]. For reference path tracking, noise suppression, nuisance rejection, and model uncertainty, an analysis was performed to compare the effectiveness of the FOFPD + FOI controller with the IOOFPD + IOI controller, the PID controller, and fractional-order proportional, integral, and derivative (FOPID) controller with the correct order. According to the simulation results, the proposed FOFPD+FOI controller outperforms PID, FOPID, and IOFPD+IOI controllers significantly. Slowtine and Li [2] created a globally stable adaptive controller by assuming full-state feedback and applying Lyapunov stability analysis. A performance comparison was conducted between fractional-order fuzzy PID (FOFPID) and integer-order fuzzy PID (IOFPID) controllers for an inverted pendulum system, which serves as the controlled plant. Four evolutionary optimization techniques (SSO, PSO, GA, and ACO) were used to fine-tune the parameters of each controller. Comparison analysis revealed that the FOFPID controller with SSO had the best time response characteristics and the least amount of tuning time [3]. The PUMA robot manipulator finds extensive use in critical areas, such as medical, automotive, education, and other fields, where human operation is deemed difficult. This robot's dimensions and parameters were all known and recorded in various literary works. [4]. The robot's performance can be classified as linear or nonlinear based on the dynamic robot model. Dynamic simulation can be used to specify specific dynamic properties related to the system's behavior, such as inertia, Coriolis, and centrifugal forces, as well as other relevant parameters. It also describes the relationship between joint movement, velocity, acceleration, and torque with current or voltage [5]. The author also suggested a method that completely forgoes the usage of observers. Four layers comprise the swarm-based metaheuristic optimization technique known as "grey wolves," introduced by Hasan et al. in 2013 [6]. This work proposes a novel direct adaptive controller that enhances an existing PID controller by utilizing fuzzy logic to account for unknown terms. Robotic manipulators are being widely used in the manufacturing sector. Their end-effectors must frequently move from one location to another and follow a predetermined trajectory. The control of robot manipulators via trajectory tracking has been the subject of numerous studies [7-9]. The majority of currently used robot manipulator systems merely employ conventional PID controllers, which are simple to use. Sliding Mode Controllers (SMCs) are the most widely used nonlinear model-based controllers and have been effectively applied in numerous applications. To achieve the desired tasks with excellent stability, the Integral Sliding Mode Controller (ISMC) was recommended for the manipulator output [10]. PSO can efficiently identify the switching sliding control and PID control, as shown by tests and simulations [11-13]. The goal of this study is to incorporate a fractional order feature into the current PID controller to enhance tracking time performance. The paper is structured as follows: Section 2 focuses on robot manipulator modeling, and Section 3 covers fractional-order strategy control. The Evolutionary Optimization Algorithm has been used to achieve optimal gains, as discussed in Section 4. Section 5 presents the simulation's findings. Finally, conclusions were drawn.

2.GENERAL SYSTEMMODEL OF PUMA 560

The following matrix equation can be used to represent the dynamic model of an n-link rigid-body robot manipulator in general [14]:

$$\mathbf{M(q)\ddot{q}+C(q,\dot{q})\dot{q}+G(q)=\tau} \quad (1)$$

where $q\in\Re n$ is the vector of angular position, $M(q)\in\Re n\times n$ is the matrix of positive definite symmetric inertia, $C(q,\dot{q})\in\Re n\times n$ is the matrix of Coriolis-Centrifugal, $G(q)$ is the vector of gravity, and $\tau \in\Re n$ is the control input vector. Eq. (1) can be rewritten in the state space model [14, 21, 23]:

$$\mathbf{\dot{x}=A(x)+B(x)\tau}$$
$$\mathbf{y=Cx} \quad (2)$$

where:

$x=[x1x2]T=[q\ \dot{q}]T$

$B(x) = \begin{bmatrix} 0 \\ M^{-1}(x) \end{bmatrix}$, $C(x)= [I\ \ 0]$

3.FRACTIONAL-ORDER PID CONTROLLER

The PID controller is one of the popular control strategies. It is used in many different contexts due to its straightforward implementation. One way to depict the PID controller is as follows [14, 15]:

$$\mathbf{u(t) = K_P\, e(t) + 1/T_i \int e(t)\, dt + T_d \dot{e}(t)} \quad (3)$$

where $u(t)$ denotes the control action, (t) denotes the control action, $e(t)$ denotes the error, and $\dot{e}(t)$ denotes the error rate of change. Kp, the proportional gain, generates a control signal proportional to the error signal $e(t)$. Td is the derivative term that generates a control signal proportional to the rate of change of the error $\dot{e}(t)$ with respect to time, leading to output overshoot damping and hence an enhanced transient response. By continuously integrating the error signal $e(t)$, the integral term Ti reduces the steady-state error [15]. In 1999, Podlubny unveiled the Fractional order PID controller (FOPID) controller. The integration and differentiation activities can be conducted in any order thanks to the FOPID controller, which is a stretch of the traditional PID controller. The equation for differential of the I^a D^b controller is expressed as [16, 22, 23]:

$$\mathbf{u(t) = K_P e\,(t) + K_i D^{-a} e(t) + K_d D^{-b} e(t)} \quad (4)$$

An ordinary integer-order PID controller was generated if a=b=1, an ordinary integer-order PD controller was generated if a =0 and b =1, and an ordinary integer-order PI controller is generated if a =1 and b =0 [17,18]. The integral and derivative of the PID plan are shown in Fig. 1, with further flexibility in modifying the FOPID controller's commands. The standard PID controller can be precisely characterized by just four points on the plane; however, the FOPID controller can be explained as the entire confined x-y plane [19].

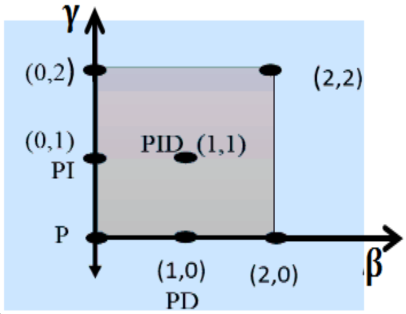

Fig. 1 PID and FOPID Order of Derivative and Integral [20].

4.EVOLUTIONARY OPTIMIZATION (EO)

Researchers using Evolutionary Optimization (EO) methods have examined a variety of wild organisms. In order to create an artificial system that functions similarly to biological entities, scientists can study how living things grow and adapt to their environment. This field of study is known as bionics or biological electronics. For instance, the invention of the airplane was inspired by the flight of birds. The radar models the bat's behavior. As a result, the methods of specific optimization algorithms are inspired by nature. The performance of the control system is enhanced in this study by applying the Grey Wolves Optimization (GWO) technique [21-23].

4.1.The Optimization Algorithm of Grey Wolves

Four layers comprise the swarm-based metaheuristic optimization technique known as "grey wolves," introduced by Hasan et al. in 2013. A conceptual program called "grey wolves' optimization" (GWO) mimics the social structure and hunting tactics of grey wolves. Grey wolves have a rigid social hierarchy and live in packs in the wild. There are usually five to twelve people in the group. The pyramid in Fig. 2 represents the hierarchical social system of grey wolves, with four tiers. At the summit of the pyramid, alpha (α) is the group leader and is responsible for decision-making and group dynamics, then beta (β) follows. Alpha wolves receive advice from beta wolves. The information is presented to alpha and beta by the third level of the pyramid, delta (δ).

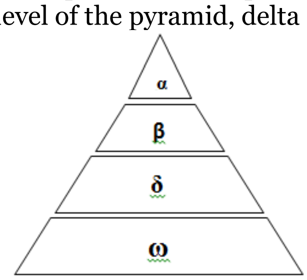

Fig. 2 Hierarchy of Grey Wolves.

The three steps in a grey wolf's hunting simulation are finding the prey, circling the prey, and attacking the prey. It can be expressed as an equation in theory. In terms of geography, the alpha, beta, and delta are more knowledgeable than the rest. As a result, the first of three options are kept. These options have identified and need the other search factors to adjust their rankings in light of the top search factor's position, which is represented as follows [24, 12]:

$$\overrightarrow{D_c} = |\overrightarrow{C_1}, \overrightarrow{X_c} - \vec{X}|,$$
$$\overrightarrow{D_b} = |\overrightarrow{C_2}, \overrightarrow{X_b} - \vec{X}|,$$
$$\overrightarrow{D_d} = |\overrightarrow{C_3}, \overrightarrow{X_d} - \vec{X}| \quad (5)$$

$$\overrightarrow{X_1} = \overrightarrow{X_c} - \overrightarrow{A_1}(\overrightarrow{D_c}),$$
$$\overrightarrow{X_2} = \overrightarrow{X_b} - \overrightarrow{A_2}.(\overrightarrow{D_b}),$$
$$\overrightarrow{X_3} = \overrightarrow{X_d} - \overrightarrow{A_3}.(\overrightarrow{D_d}) \quad (6)$$

$$\vec{X}(t+1) = \frac{\overrightarrow{X_1} + \overrightarrow{X_2} + \overrightarrow{X_3}}{3} \quad (7)$$

The vectors A and C can be determined as follows:

$\vec{A} = 2\vec{a}, \overrightarrow{r_1} - \vec{a}, \vec{C} = 2, \overrightarrow{r_2}$

4.2. Controller Parameters Tuning Using GWO Technique

To achieve optimal performance and improve system stability, the settings of the suggested D^{-a} and D^{-b} controllers applied to the Puma 560 system should be adjusted. The suggested controller's design process is automated with the help of the GWO. In addition to the fractional order of the integrator and differentiator, the gains of the controllers are the parameters that need to be adjusted. The Integral Time Absolute Error (ITAE) performance index, which is stated as follows, is minimized by the GWO algorithm [25, 13]:

$$ITAE = \int_0^{\infty} t|e(t)|dt \quad (8)$$

The suggested controller that was employed has the following GWO algorithm configuration setting:

- The population is fifty.
- There have been 100 iterations.
- Tuning parameter count: (12) varies according to the controller.
- 0 to 40 is the search space for a gain of the controller.
- 0 to 1 is the search space for parameters of fractional order (ε and ε).
- Value of the coefficient (a): 0 to 2.

The flowchart in Fig. 3 illustrates how the parameters for the two loops of the D^{-a} and D^{-b} are adjusted using the GWO algorithm. The author created the flowchart using the algorithm code as a guide.

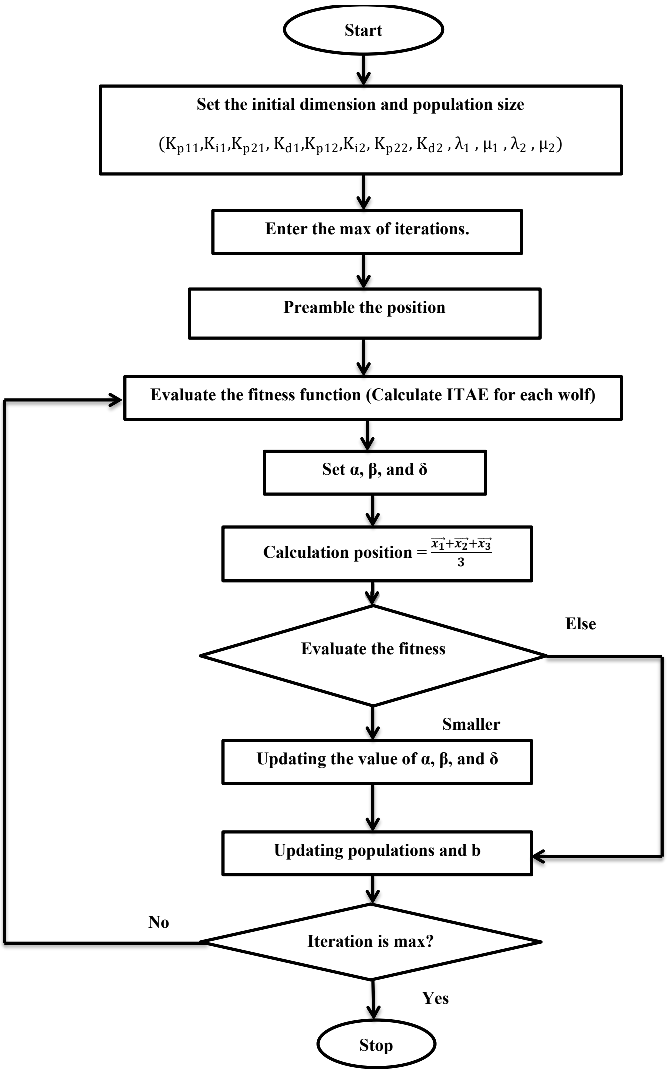

Fig. 3 Flowchart for Tuning the Proposed Controller Parameters Using GWO.

5.COMPUTER SIMULATION RESULTS

The Puma robot (PR560) represents a highly nonlinear and complex system. It needs to control the end-effector (last joint) for several scenarios. The best method for testing PR560 is to use a simulation computer. The simulation was achieved using a combination of MATLAB code and Simulink version 2020. Several cases are used to test PR560 with the suggested controller and compare it with related works. The proposed controller is a fractional-order system with PID characteristics developed using the Grey Wolf optimization algorithm. Figure 4 illustrates the structure of FOPID with GWO for simulating the Puma 560.

Case 1:-Optimized F-PID.

Figure 5 shows the time response of PR560 when using an optimized F-PID controller for step input and no load in the end effector. The rise time was 0.4 sec, and the settling time was 0.55 sec, with a maximum overshoot approaching zero.

Case 2:-Optimized Load F-PID

Figure 6 shows the time response of PR560 when using an optimized F-PID controller for a step input and maximum load (950) grams in the end effector. The rise time was (2.11) sec, and the settling time was (2.95) sec, with a maximum overshoot approaching zero.

Case 3:-Optimized Load F-PID and I-PID

Figure 7 shows the time response of PR560 when using optimized F-PID and I-PID controllers for step input and maximum gram in the end effector. The numerical results for F-PID showed that the rise time was (2.11) sec, and the settling time was (2.95) sec, with a maximum overshoot approach to zero. For integer PID, the rise time was (3.01) sec, and the settling time was (3.45) sec, with no maximum overshoot.

Case 4:-Optimized Load I- PID

Figure 8 shows the time response of PR560 when using optimized I-PID controllers for sine wave input and maximum load in the end effector. The error between the reference signal and I-PID was (0.25) at (15) sec. Figure 9 shows the time response of PR560 when utilizing optimized I-PID controllers for a sine wave input and maximum load in the end effector. The error between the reference signal and F-PID was close to zero; however, some errors were found in the beginning (0.005) at rang time (2-7) sec.

Case 5:- Comparison With Related Published Works.

Figure 10 compares F-PID, backstepping, and Fuzzy controllers with step input. According to the numerical results, the fractional order PID was superior to other in time response parameters rise time, settling time, and maximum overshoot for F-PID (30, 30.01, and 0%), backstepping controller (11.5, 30.12, and 15.5%), and Fuzzy controller (11.01, 61.23, and 27.5%), respectively. Figure 11 shows the three-dimensional circular shape of the proposed controller-optimized F-PID, demonstrating a complex task that ensures high accuracy and efficiency.

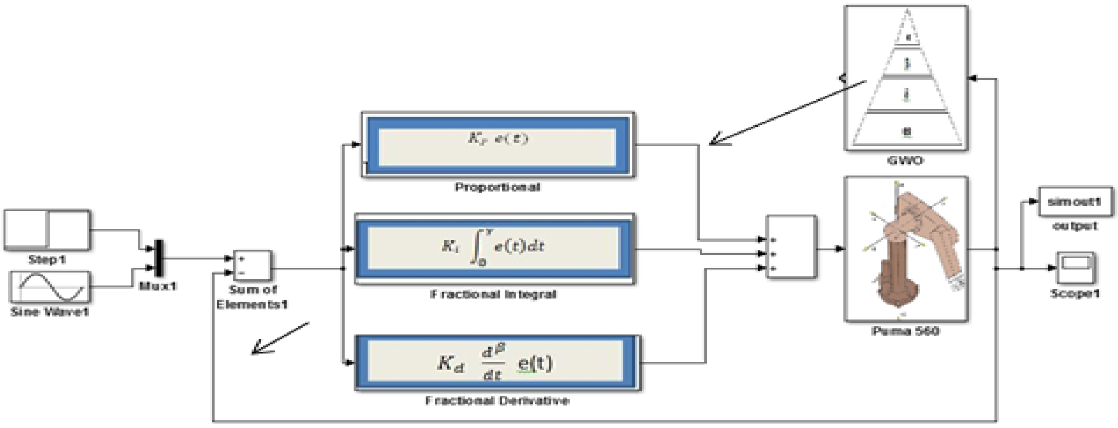

Fig. 4 The Structure of Fractional-Order PID for Puma 560 Based on GWO.

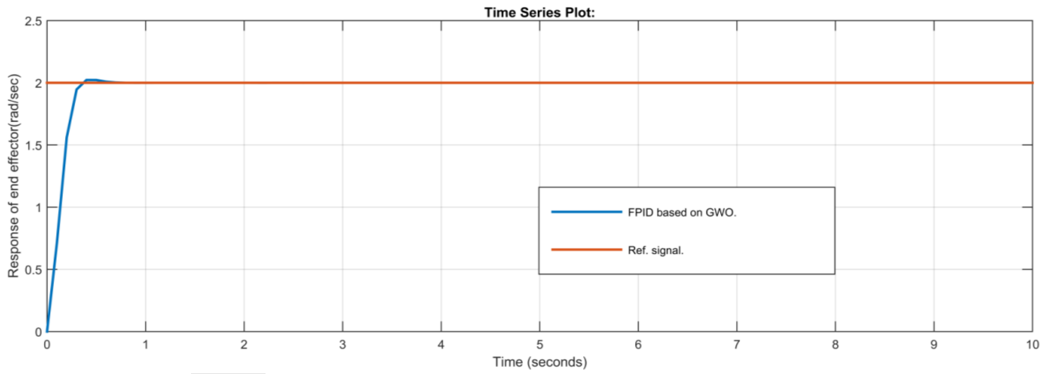

Fig. 5 Step Response Time of End–Effector with no Load.

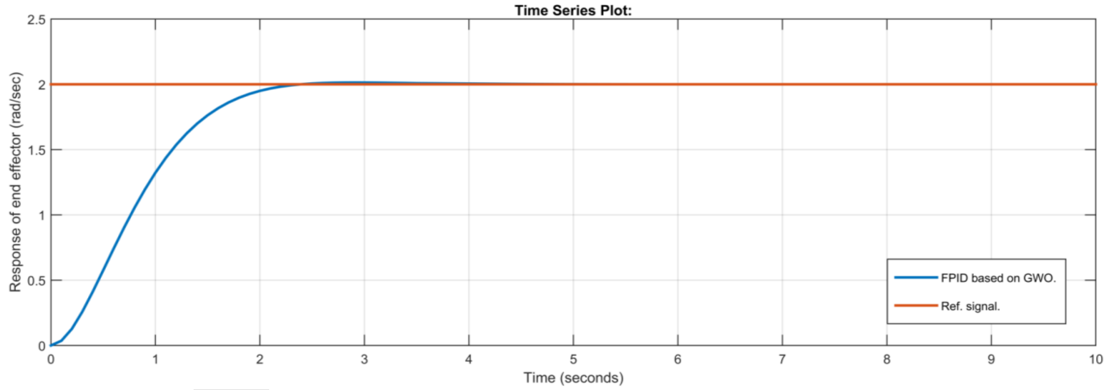

Fig. 6 Seep Response of End–Effector with Maximum Load.

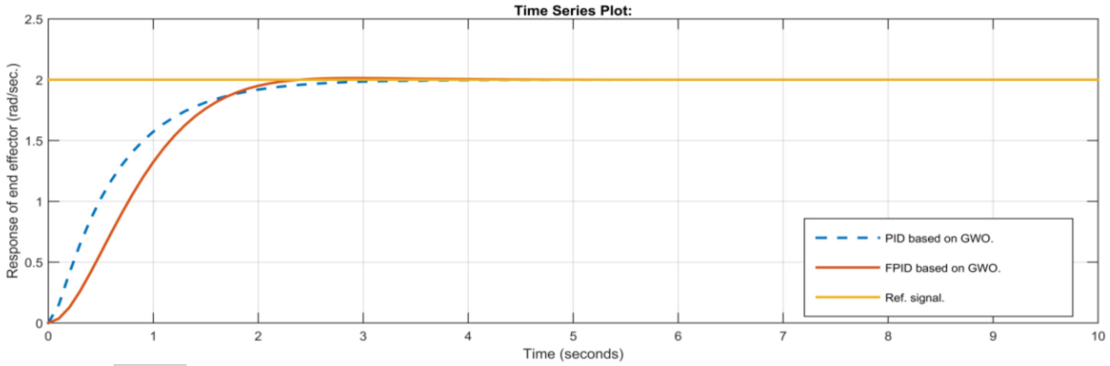

Fig. 7 Comparison of Response Time between FPID and I-PID Controllers.

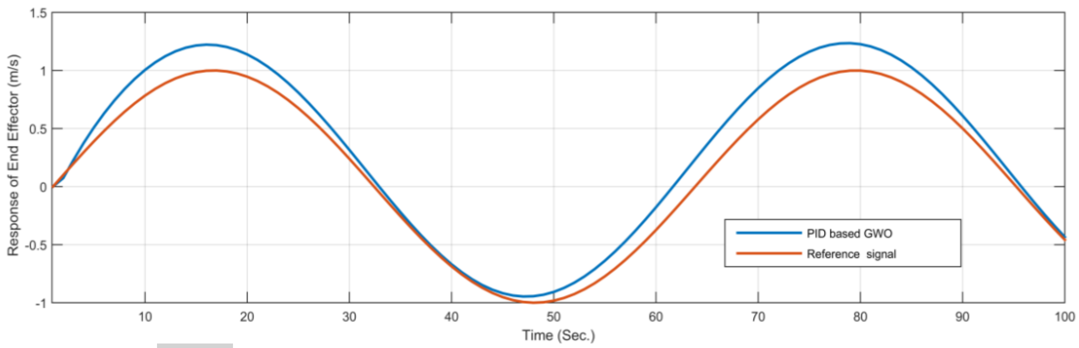

Fig. 8 Response Time for PID Controller when Sine Wave Input.

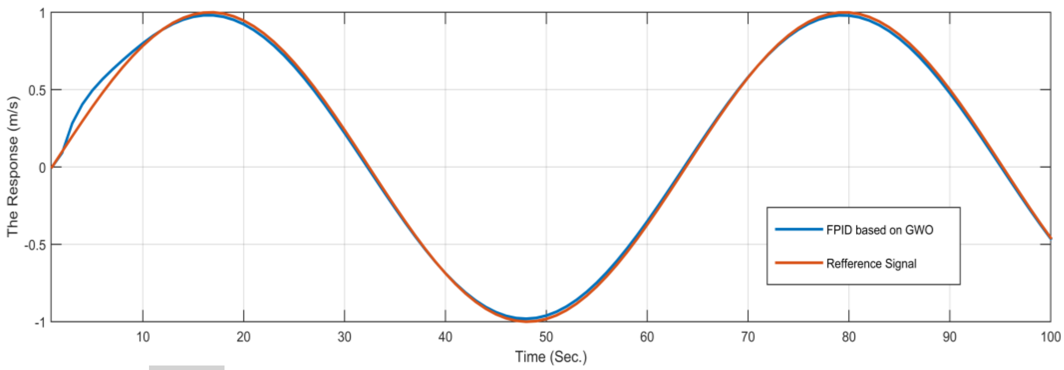

Fig. 9 Response Time for FPID Controller when Sine Wave Input.

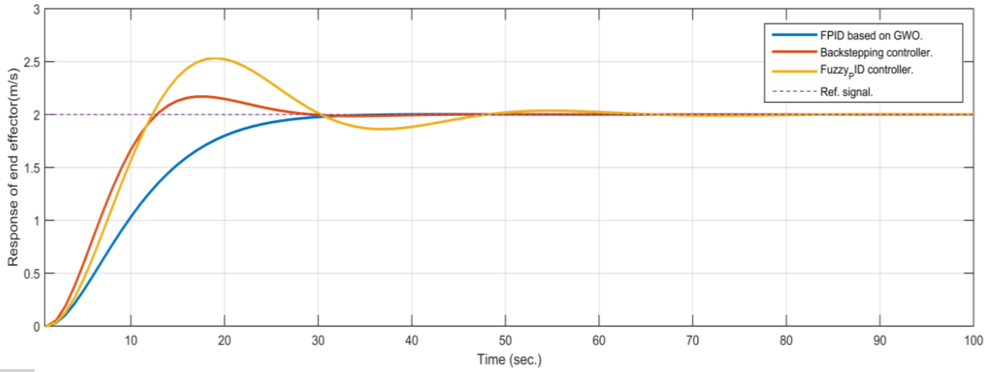

Fig. 10 Comparison Time Responses for F_PID controller Against Published Related Works.

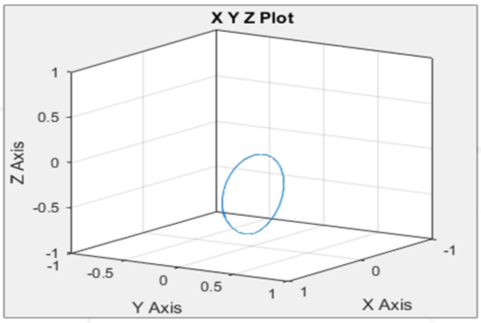

Fig. 11 3D Circle Shape by Using F_PID Controller.

6.CONCLUSION

The performance of classical PID controllers, whether with or without the existence of disturbance or load, is inaccurate. Therefore, the control is very challenging, and the nonlinear systems cannot controlled by the PID controller. Therefore, the fractional order PID controller based on grey wolf optimization can be used to avoid falling into this problem. When adding the disturbance or load, the time response for the tracking step input signal increased in the PID. In the state of using the F-PID, the time response was very good during the path. When the load was added, it was observed that it affected joint 5 or increased the end effector and the tracking error in joint 5 in PID. However, with the F-PID controller, the end effector /joint 5 achieved good performance during the path. From the simulation, it was observed that the performance of the fractional order PID with GWO disturbance/load was superior to that of another PID with GWO performance for disturbance/load to control the position of the Puma 560 robot manipulator in terms of position tracking error. The present study included multiple scenarios with the maximum load as a disturbance, including 3D plotting, sine wave testing, and step testing. The results introduced the best numerical values and very good movement to draw circular shapes. Finally, these results were compared with related published works and proved the superiority of the proposed controller over others. With this design approach, the simulation results for the Puma robot manipulator showed excellent tracking response in the operational space.

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