



College of Engineering

ISSN: 1813-162X (Print) ; 2312-7589 (Online)

Tikrit Journal of Engineering Sciences

available online at: <http://www.tj-es.com>

**TJES**  
Tikrit Journal of  
Engineering Sciences

Muhsin IA, Shwikh AM. Effect of the Gyroscopic moment, Centrifugal Force and Hydroplaning on the Critical Speed of the Vehicle. *Tikrit Journal of Engineering Sciences* 2019; 26(2): 39-48.

**Ibrahim A. Muhsin**  
**Abdallah M. Shwikh\***

Department of Mechanical  
Engineering, College of  
Engineering, Tikrit University,  
Tikrit, Iraq.

# Effect of the Gyroscopic Moment, Centrifugal Force and Hydroplaning on the Critical Speed of the Vehicle

## ABSTRACT

Driving vehicles on curved roads is dangerous because of the risk of accidents. This is due to the centrifugal force, gyroscopic moment and hydroplaning of the vehicle, ending with vehicle slipping or tipping. The aim of this research is to find the critical speed under any one of the above mentioned risks. The water pressure under the vehicle tires was calculated using Matlab R2017a in order to find the pressure value that able to lift the vehicle causing slipping and then going out of control. The effect of many parameters, on the vehicle hydroplaning have been studied. These parameters are tire width, wheel load and water layer thickness. While for vehicle slipping due to the centrifugal force or the gyroscopic moment, the following parameters have been studied. These parameters are height of vehicle gravity center, vehicle width, radius of the circular path, and track angle. The results showed that the gyroscopic torque negatively affects the critical velocity of the vehicle, and it reduced it about 0.549%. The centrifugal effect is the has the greatest influence on the gyroscopic effect, and the gyroscopic effect pushes the vehicle outward and increases the radius of the rotation, while the gyroscopic couple effect ,at low radius. of low rotation, is very small. Gyroscopic impact is increased by increasing the radius of the rotation path. The result also showed the increase of the road surface angle and surface at the turning the influence positively on the safe speed of the vehicle at all the above variables.

© 2019 TJES, College of Engineering, Tikrit University

DOI: <http://dx.doi.org/10.25130/tjes.26.2.06>

## Keywords:

Hydroplaning  
Critical speed  
Vehicle tipping  
Vehicle dynamic

## ARTICLE INFO

### Article history:

Received: 30 Sep. 2018  
Accepted: 25 Feb. 2019  
Available online: 05 Apr. 2019

دراسة تأثير العزم الجيروسكوبي وقوة الطرد المركزي والانزلاق المائي على السرعة الحرجة للمركبة

## الخلاصة

سير المركبات على الطرق المنحنية والمنعطفات ينطوي على الكثير من المخاطر التي قد تؤدي بها إلى الانزلاق أو حتى الانقلاب وذلك بسبب تأثير العزم الجيروسكوبي و قوة الطرد المركزي وخطر طوفانها بسبب طبقة الماء التي تغطي سطح الطريق . لذلك اهتمت الدراسات والبحوث بإيجاد السرعة الحرجة الأمانة للمركبة تجاه تلك المخاطر. تم احتساب قيمة ضغط الماء اللازم لرفع اطار المركبة عن سطح الطريق وقد تم الأخذ بنظر الاعتبار عوامل الطريق والمركبة المؤثرة على قيمة ذلك الضغط منها ، عرض الاطار ، وحمل الاطار ، وسمك طبقة الماء على سطح الطريق. كما تمت دراسة تأثير العوامل التي تؤثر في العزم الجيروسكوبي وقوة الطرد المركزي ، وهي ارتفاع مركز الثقل للمركبة ، وعرض المركبة ، ونصف قطر دائرة انعطاف الطريق وزاوية ميلان سطح الطريق. أظهرت النتائج أن العزم الجيروسكوبي يؤثر سلبا على السرعة الحرجة للمركبة ، ويكون قليل حيث لا يتجاوز (0.549%) ويكون لتأثير الطرد المركزي التأثير الأكبر والتي تغطي على التأثير الجيروسكوبي ، ويزداد ظهور التأثير الجيروسكوبي بزيادة نصف قطر مسار الدوران، ويكون لزيادة عرض المركبة وارتفاع مركز ثقلها تأثير واضح على زيادة هذا التأثير. كما أظهرت النتائج أن لزيادة ميلان سطح الطريق عند الانعطاف تأثير إيجابي على السرعة الأمانة للمركبة عند جميع المتغيرات أعلاه.

\* Corresponding author: E-mail : [abdsadman@gmail.com](mailto:abdsadman@gmail.com)

## 1.Introduction

Maintaining vehicle stability during driving on roads is an important matter, therefore the researchers developed and worked many concepts and methods of multiple innovative ideas for dealing with this matter. All these prediction techniques were used to monitor early vehicle accident, to come up with a new design criteria for the vehicle that involving all this parameters, in this prediction calculations. Which means redesigning the vehicle with the consideration of stability improvement, and achieving a higher speed without accidents due to any of the mentioned reason. These accidents could happen due to skewed maneuver or because of the rapid loss of friction on the roads and many other reasons. The presence of a water layer on the road surface cause losing the vehicle control also. This would happen when the force resulting from the water pressure is equal to the vehicle load including its mass so-called water slide [1]. Many researches and studies that dealing with this matter, has been conducted since the sixties of the last century. This has been done in order to find the safe vehicle speed and safe aircraft landing speed, on the runways. There are many factors that may effect the critical speed of the vehicle including the characteristics of tires, vehicles, environmental conditions and driver skill , and this could occur on uneven roads, cliffs or winding roads, or traveling on a curved roads and therefore an angular momentum will be initiated producing an effective gyroscopic torque that have what called gyroscopic effect, which is exist in a large scale In the air or marine vehicles such as aircraft and ships. In addition to that a centrifugal force will be exist which make the situation more dangerous.

The gyroscope is a device in which the axis of spinning wheel or disc gives an angular motion around the an axis, perpendicular to the axis of spin, an angular acceleration acts on the body about the third vertical axis. while rotating ,orientation of axis of spinning wheel is unaffected by the external tilting or rotation of the mounting foundation. Applications of gyroscopes include inertial navigation systems as in the Hubble telescope, intercontinental ballistic missiles and the stabilization of the flying vehicles unmanned aerial vehicles, and commercial ships [2].

Many studies used the historical accidents data and took different kinds of influencing factors into account as an approach to analyze the changes in safety levels of signalized intersections after a specific countermeasure is implemented. However, it takes a very long time to obtain statistically sufficient data. Brach [3] investigated the speed at which the vehicle deviates from its trajectory through the critical velocity equation that takes into account the centrifugal effect that depends on the radius of the circular path. Chang [4] study the stability of vehicles on the horizontal plain roads and the relationship between the minimum radius of the road

with the lateral friction and the angle of the road hopes at a specific design speed, taking into account the vehicle's body, and found approximately 12-30% difference in the values of the minimum radius, And that is the coup may be more important than the lateral slip in modern vehicles. Lambert [5] Checked of the most effective characteristics on the vehicle, in coup and tested the methods of the electronic stability controllers to improve stability, and develop a detailed model of the vehicle including the tilt vehicle and the dynamics of individual wheels and the development of equations to calculate the maximum side acceleration and allowed speed before the coup, and proved that the results can improve stability and avoid the coup With proper implementation of ESC and output limits. Metz. [6] Studied the vehicles sliding during uniform rotation and through the experience of movement of vehicles under wet conditions road. The possibility of the vehicle's sliding is usually caused by several reasons, including movement of the vehicle through braking or changing the speed of the vehicle and other reasons, (Wet), and the interference in the frame and sliding behavior was studied theoretically and accurately. Muhsin et al. [7] studied the safe speed of the vehicles on the wet road, taking into consideration two types of tires, smooth tire, and decorative tire and studied the effect of several factors, including the width of the tire, radius and the number of grooves, width , depth, shape ratio, tire load and thickness of the water layer at critical speed, they concluded that increasing the tire width and radius, thickness of the water layer on the road negatively effects the critical speed of the vehicle. Increasing the tire load and the number of grooves, depth, width and shape ratio positively effect the critical speed of the vehicle. Žuraulis et al. [8] The analysis of the lateral dynamic factors that effecting the vehicle, the angle of turning of the wheel to determine the critical velocity values in which the control of the vehicle is lost. The experimental results were compared with the theoretical calculations, at 10 m path radius, the experimental speed was 27.86 km / km / h. 30.57. Nishad Kumbhojkar et al. [9] worked on introducing the gyroscopic effect on a two-wheeled vehicle to achieve stability on the road. On the basis of the gyroscope on ships balance and aircraft. They created a two-wheeled model containing rotary disks that would introduce the gyroscopic effect to achieve balance and stability. Colvin [10] developed the stability control of gyroscopic torque (CMG) and verified it using the MATLAB / Simulink program, thus setting a

model for mentoring behavior and response to an unstable body with gyroscopic installation application. In order to improve the safety of stability of the two-wheel vehicles, which will provide the ability to maneuver and stabile of its users compared to vehicles. Implementing a vehicle using this technique can generate a safer and more maneuverable vehicles for general and military users. Luo et al. [11] studied the risk of the water slide on horizontal and vertical slopes and calculate the predicted velocity of water slide using two models that based on Gallaway and USF using three-dimensional laser imaging data and showed that the sliding speed is more sensitive to the lateral slope of the longitudinal slope, decreases with longitudinal grade increase and increases with lateral slope. Mashadi et al. [12] designed a gyroscopic system to apply reverse (corrective) torque to the vehicle in the opposite direction of the coup. Its performance has been verified during simulations of some intense maneuvers and the results showed that the system is able to install the car successfully. Gopinath et al. [13] used the same principle as gyroscopic stabilizers used in ships and aeronautical aircraft, he designed and manufactured the self-balancing of a two-wheeled vehicle. The study was carried out by taking various factors into account, and optimal balance results were obtained with the effect of the gyroscopic torque. The main purposes of this study to find the effect of hydroplaning ,centrifugal force and gyroscopic moment. separately on critical speed of the vehicle, and find the effect of hydroplaning, centrifugal force and gyroscopic moment. to gather on critical speed of the vehicle

## 2.RESEARCH METHODOLOGY

A mathematical model of a standard four wheelers under curved navigation was considered with all the forces and torques encounters. This model was imported in MATLAB and a standard set of vehicle parameters were assumed for analysis of the model. The system was analyzed for the dependency and effect of each parameter on the stability of the four wheeler during cornering factors and parameters governing the four wheeler dynamics during curve navigation were identified, by dynamically nullifying instabilities in the vehicular system .

## 3.MATHEMATICAL MODELLING

### 3.1.Modeling Vehicle Coup due to Gyroscopic Moment Effect

When the vehicle is turn on road, three loads effect it, as the weight of the vehicle, centrifugal force and reactive gyroscopic moment. The vehicle weight effect vertically through the center of gravity. While the effect

of centrifugal force appears when the vehicle travelling on a circular path, this effect is radially outward through the center of gravity, and it overturn the vehicle. the centrifugal force is:

**Table 1**  
Parameters used in the analysis

Parameter	Details
Passenger car tire	ASTM E 501 standard G78-15 tire
Analyzed tire	Smooth tire
Pavement surface	Smooth plane surface
Vehicle load	1600 , 2000 , 2400 and 2800 KG
Wheel load	400 , 500 , 600 and 700 KG
Tire width	140 , 160 , 180 , 200 and 220 mm
Cross Section Radius	280 , 300 , 320 , 340 and 360 mm
Water depth	5 , 7 , 8 and 10 mm
Vehicle width	1.5 , 1.6 , 1.7 , 1.8 and 1.9 m
Height of the gravity center of the vehicle $H_{C.G}$	0.5 , 0.6 , 0.7 and 0.8 m
Circular path radius	0 , 50 , 100 , 150 , 200 and 240 m
Track angle of the road	0 , 2.29 , 3.39 and 5 degree

$$F_c = M\omega_p^2 R = M \frac{v^2}{R^2} R \quad (1)$$

$$F_c = \frac{MV^2}{R} \quad (2)$$

$$F_c = \frac{MV^2}{R} \quad (2)$$

While the reactive gyroscopic moment effect due to rotating part in vehicle, effect while turning on a circular path. To learn the mechanism of the reactive gyroscopic moment. Take a reference frame with the axes  $g_1$ ,  $g_2$  and  $g_3$  is fixed to the gyroscope as the Fig. 1.a These axes are initially parallel to the axes X, Y and Z respectively fixed to the vehicle. The Kane–Levinson method is applied for deriving the gyroscope equations of motion according to [14]:

$$M_{GX} \sin \sin \theta + M_{GY} + I_R(-\ddot{\theta} \sin \sin \theta + \dot{\theta} \dot{\phi} \cos \cos \theta + \ddot{\theta}) = 0 \quad (3)$$

$$M_{GZ} \sin \sin \theta + \frac{I_R}{2} (-\ddot{\theta} - 2\dot{\Omega}\dot{\theta} \cos \cos \theta + \dot{\theta}^2 \sin \sin \theta \cos \cos \theta) = 0 \quad (4)$$

$$\begin{aligned} M_{GX} M_{GY} \sin \sin \theta \\ - \frac{I_R}{2} [\ddot{\theta} \cos^2 \theta - 2\dot{\Omega}\dot{\theta} \cos \cos \theta \\ + 2\dot{\Omega}^2 \sin^2 \theta + \dot{\theta} \dot{\theta} \sin \sin (2\theta)] \\ = 0 \end{aligned} \quad (5)$$

The gyroscope rotor rotates permanently around its spinning axis with a constant speed. In this case,  $M_{GY}$  is due to energy dissipation in the bearing and is sufficiently small to be eliminated. The gyroscope produces a torque when a precession occurs or vice versa. Making a revolution about  $g_3$  causes a torque  $M_{GX}$  about  $g_1$ , which can stabilize the vehicle during rollover. On these assumptions, the gyroscope equations are reduced to [14]:

$$M_{GX} = \frac{I_R}{2} [\ddot{\theta} \cos^2 \theta - 2\dot{\Omega}\dot{\theta} \cos \cos \theta + \dot{\theta} \dot{\theta} \sin \sin (2\theta)] = 0 \quad (6)$$

$$M_{GZ} = \frac{I_R}{2} (\ddot{\theta} + 2\dot{\Omega}\dot{\theta} \cos \cos \theta - \dot{\theta}^2 \sin \sin \theta \cos \cos \theta) = 0 \quad (7)$$

**Size of gyroscope.** The rotor dimensions are an important part of the gyroscope. The spin speed of the rotor is high and, according to Eq. 6, other terms can be ignored to obtain a relation that enable us to estimate the rotor dimensions [12], which is:

$$M_{GX} = I_R \dot{\Omega} \dot{\theta} \cos \cos \theta \quad (8)$$

When the vehicle is moving along a curved path, The direction of torque vector will be along the rotating spin vector, 90 degree in the direction of precession vector. Due to this, the reactive gyroscopic couple is applied by the axle and wheels, The four wheels and engine are considered two different sets of rotors since they are rotating with different speed and may be in different directions. In this studied neglecting the reactive gyroscopic moment due to engine.

The reactive gyroscopic moment due to four wheels are:

$$M_{GX} = 4I_R \dot{\Omega} \dot{\theta} \quad (9)$$

Where:

$$M_{GX} = \tau, \dot{\Omega} = \omega, \dot{\theta} = \omega_p$$

the governing equation during the vehicle motion of where rolling not exist as the Fig.1.b is:

$$\Sigma M = 0 \quad (10)$$

This leads to:

$$\tau = 2R_i + Mg \frac{a}{2} + F_c H_{c.g} \quad (11)$$

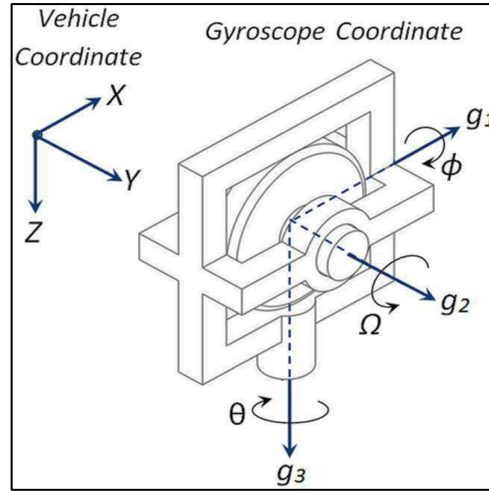


Fig. 1.a. Gyroscope and vehicle coordinate system [12]

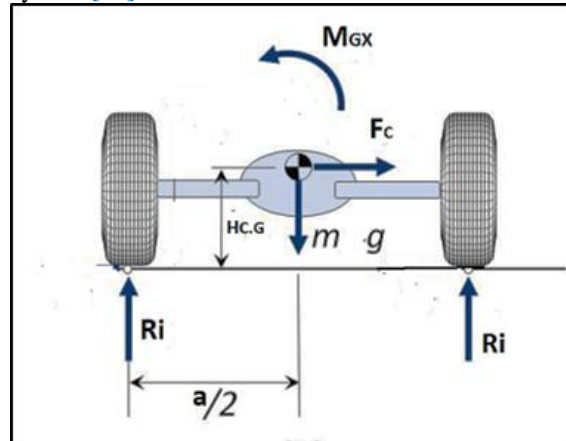


Fig. 1.b. Rear view the non-rolling of the vehicle [12].

### 3.2. Modeling of the vehicle hydroplaning

Reynolds equation has been used to analyses the parameters and asses the performance of the operation. The contact region between the tire and the surface road looks like the convergence zone in the conventional hydrodynamic bearing, therefore Reynolds equation [14] has been used :

$$\frac{\partial}{\partial x} \left( \frac{h^3}{\mu} \frac{\partial P}{\partial x} \right) + \frac{\partial}{\partial z} \left( \frac{h^3}{\mu} \frac{\partial P}{\partial z} \right) = 6U \frac{\partial h}{\partial x} \quad (12)$$

Considering the viscosity is constant over the whole contact region ,then expanding Reynolds equation and then using finite difference method of five nodes this yield this form of equation :

$$P_{i,j} + C_1 P_{i+1,j} + C_2 P_{i-1,j} + C_3 P_{i,j+1} + C_3 P_{i,j-1} = C_4 \quad (13)$$

This equation has been solved numerically using a computer program utilizing the matrix method Gaussian elimination ( MATLAB R2017a) in solving the simultaneous equations.

The force that effect the tire at any node Fig. 2 [7] is :

$$F_N = \sum_{j=1}^{j=n} \sum_{i=1}^{i=m} P_{i,j} * b * \sin \sin \left( \beta - \frac{X}{r} \right) \quad (14)$$

$$F_T = \sum_{j=1}^{j=n} \sum_{i=1}^{i=m} P_{i,j} * b * \cos \cos \left( \beta - \frac{X}{r} \right) \quad (15)$$

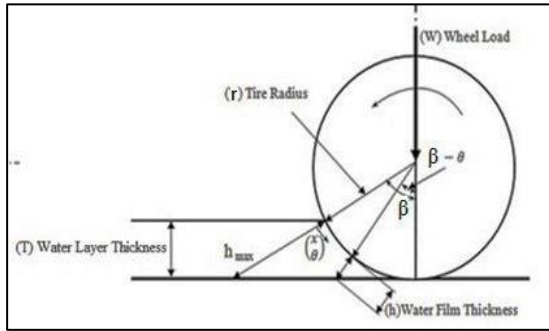


Fig. 2. Tire geometry [7].

### Vehicle stability during turning

#### 1- safe speed of the vehicle regarding hydroplaning

At a horizontal plane where the track angle equal zero, equation of safe speed equal :

$$F_{up} \geq W * g \quad (16)$$

Where the track angle equal ( $\theta$ ) could be any value as shown on figure 3 ,then equation of safe speed equal to :

$$F_{up} = F_N \times \cos \cos \theta + F_T \times \sin \sin \theta \quad (17)$$

$$F_{up} \geq W * g * \cos \cos \theta + \frac{F}{4} * \sin \sin \theta \quad (18)$$

#### 2- safe speed of the vehicle regarding coup, due to gyroscopic moment effect.

During navigation on a curved road then active gyroscopic couple would be created, acting on the

system due to the rotating parts, and also centrifugal couple has a tendency to overturn the vehicle , therefore , the total overturning couple ( $\tau_o$ ) is given by:

$\tau_o$ =active gyroscopic couple + centrifugal couple  
 $= \tau + \tau_c$

$$\tau_w = W_v * \frac{a}{2} \quad (19)$$

At a horizontal plane , where the track angle equal zero , then the above equation equal is rewritten as follow [9] :

$$\tau_o \geq \tau_w$$

$$\frac{W_v * V^2 * H_{c.g}}{R} + \frac{4I_R V^2}{R * r} \geq W_v * g * \frac{a}{2} \quad (20)$$

Where the track angle equal  $\theta$  as shown on figure 4 :

$$\begin{aligned} & \left( \frac{W_v * V^2 * H_{c.g}}{R} + \frac{4I_R V^2}{R * r} \right) * \cos \cos \theta \\ & - (W_v * g * H_{c.g} * \sin \sin \theta) \\ & \geq \left( \sin \sin \theta * \left( \frac{W_v * V^2}{R} \right) \right. \\ & \left. + W_v * g * \cos \cos \theta \right) \\ & * \frac{a}{2} \quad (21) \end{aligned}$$

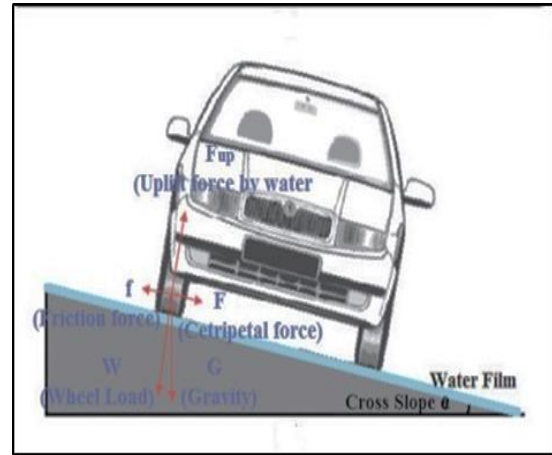
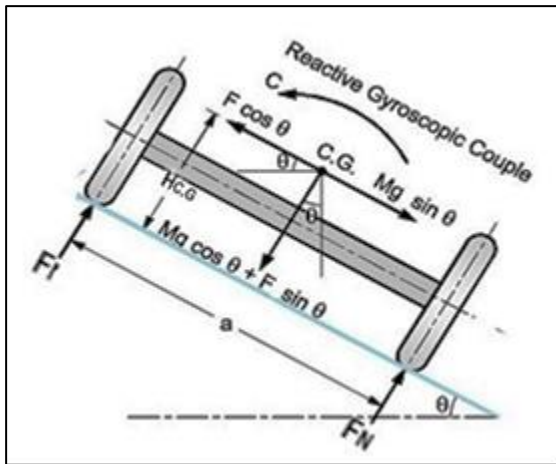


Fig.3. Vehicle travelling on pavement slippery segment with horizontal curve [11]



**Fig. 4.** Couples and force acting on a vehicle a during curved negotiation

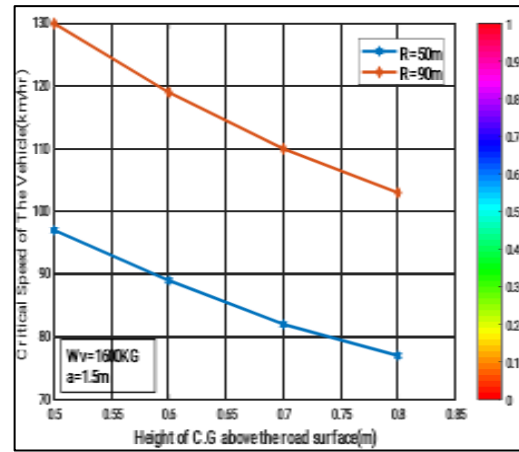
#### 4.Results and Discussion

The computer program results had been drawn on (17) figures. these figures may be divided into three separate groups. These groups are:

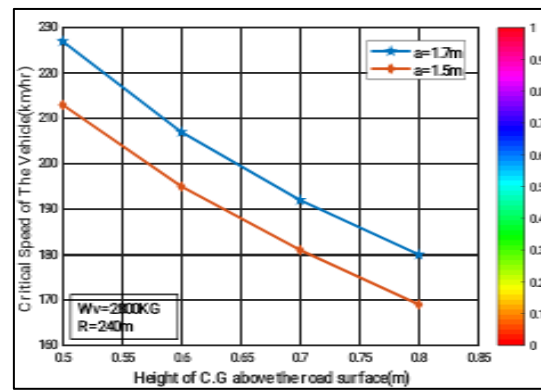
- A.Critical speed depends on the rules of the gyroscopic torque only..
- B.Critical speed depends on the rules of the gyroscopic moment in addition to the centrifugal force effect , both together.
- C.Critical speed depends on the rules of the hydroplaning only.

##### Gyroscopic effect

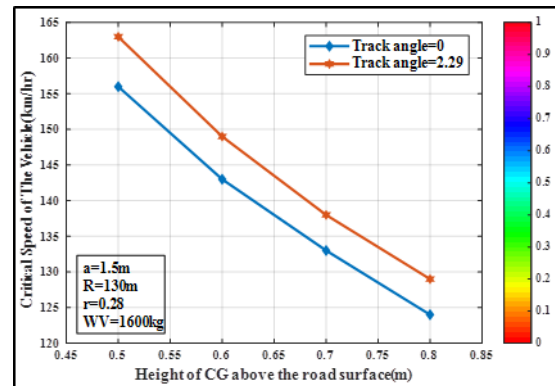
Figs 5,6,7,8,9,10,,11,12 and 13 show the relation between the critical speed , due to gyroscopic moment, with different vehicle and road parameters. For fig 5,6 and 7 shows an inverse proportional relation between the critical speed and C.G, for different values of R, a and track angle. while figs 8,9,10,11,12 and 13 show a proportional relation between the critical speed and different vehicle and road parameters. Through these figures it is noticed that the increase in the height of the center of gravity negatively affects the critical speed and reduces this effect by increasing the radius of the track and increase the width of the vehicle and reduce this effect by increasing the angle of the track road. This confirms that high-speed vehicles (such as racing cars) have a low gravity center height of gravity and large width as the vehicle's design allows. All the mentioned figures are in full agreement with those obtained by: the following researchers : Lambert [6], Zuraul et al. [8], Akshey khot [9], Colvin [10], Mashadi et al. [12].



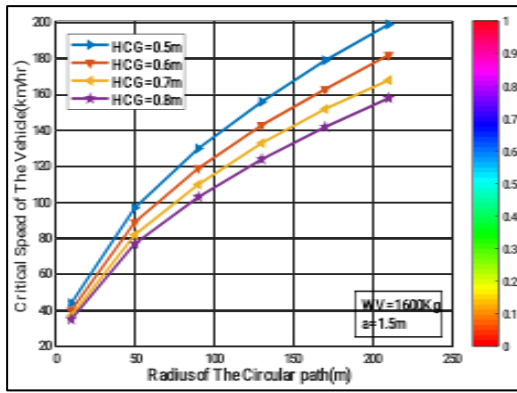
**Fig. 5.** The relation between Critical Speed and HC.G for different Path Radius.



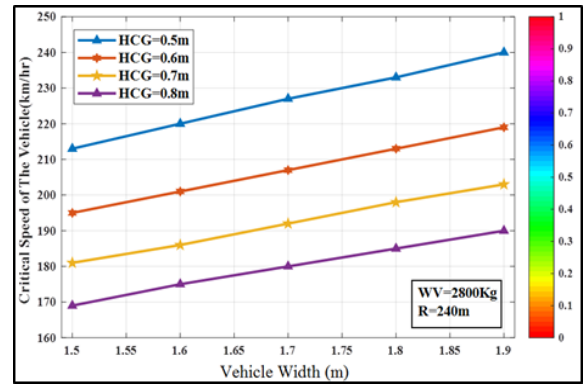
**Fig. 6.** The relation between Critical Speed and  $H_{C.G}$  for different vehicle Width



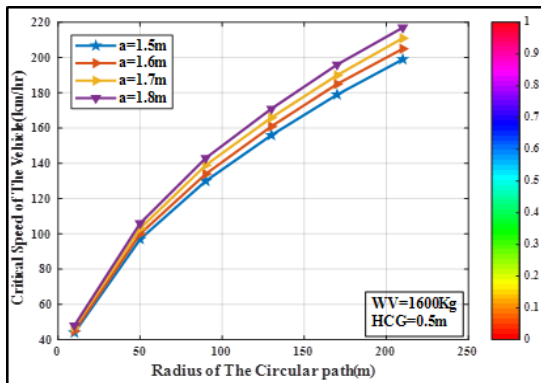
**Fig. 7.** The relation between Critical Speed and  $H_{C.G}$  for different Track angle.



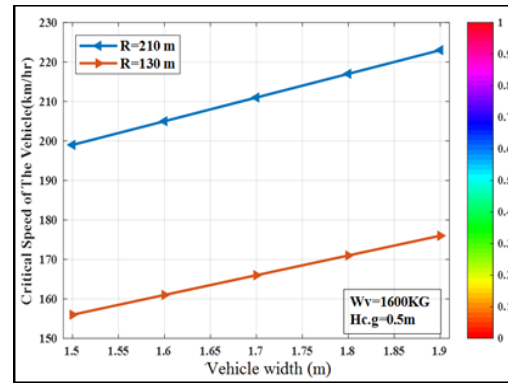
**Fig. 8.** The relation between Critical Speed and Radius of the circular path for different HC.G.



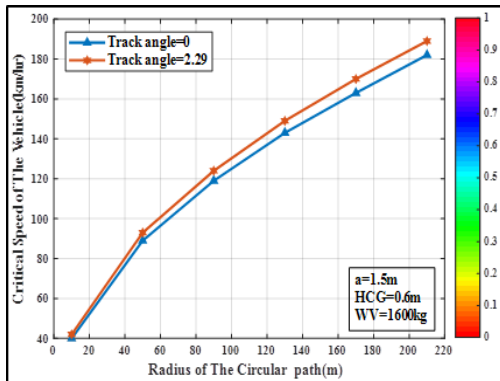
**Fig. 11.** The relation between Critical Speed and vehicle Width for different H<sub>C.G.</sub>.



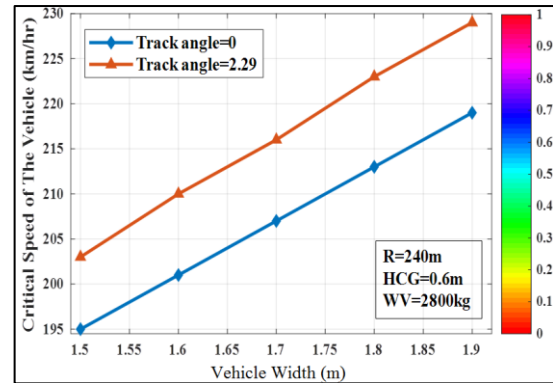
**Fig. 9.** The relation between Critical Speed and Radius of the circular path for different vehicle width.



**Fig.12.** The relation between Critical Speed and vehicle Width for different Radius of the circular path.



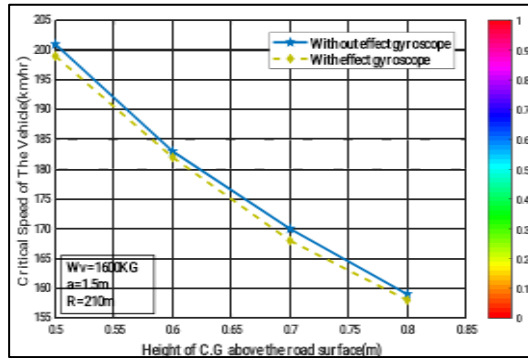
**Fig. 10.** The relation between Critical Speed and Radius of the circular path for different Track angle.



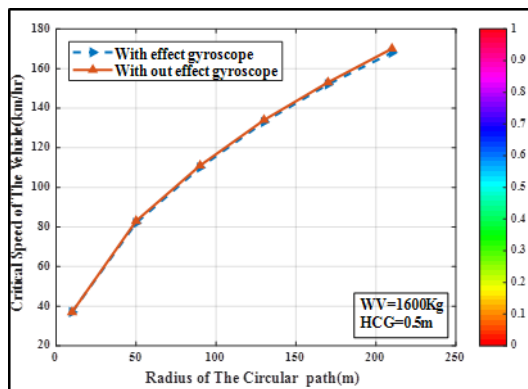
**Fig. 13.** The relation between Critical Speed and vehicle Width for different track angle.

## Gyroscope and centrifugal effect together

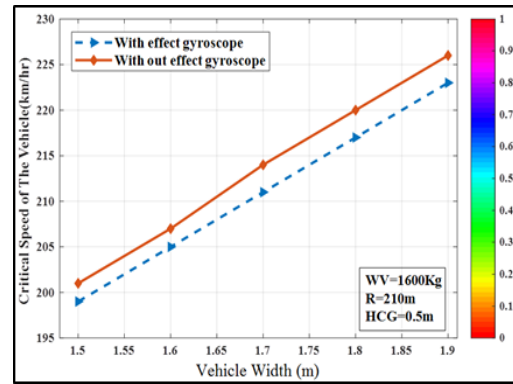
critical speed that counted on the bases of the gyroscopic moment in addition to the centrifugal force effect , both together. Figs 14,15 and 16 shows the relation between the critical speed and HC.G , R and vehicle width . The results showed that the effect of the gyroscopic couple negatively effects the safe speed of the vehicle. And increasing the width of the vehicle and its height center of weight of these vehicle a clear effect on this mater, while the effect of the gyroscopic couple at low radius of low rotation is very small. Gyroscopic impact is increased by increasing the radius of the rotation path. These figures show a curve shape and trends in a satisfactory agreement with those obtained by: Brach [3], Chang [4] and Lambert [6].



**Fig.14.** The relation between Critical Speed and HC.G with effect and out effect gyroscopic.



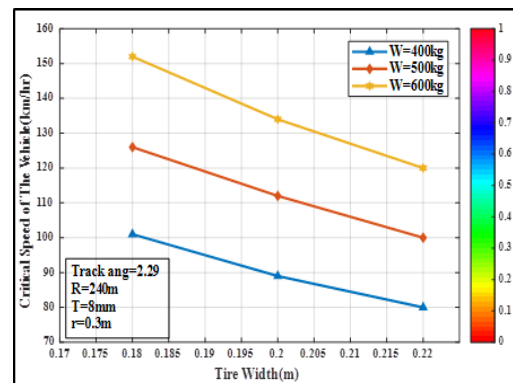
**Fig. 15.** The relation between Critical Speed and Radius of the circular path with effect and out effect gyroscopic.



**Fig. 16.** The relation between Critical Speed and vehicle width with effect and out effect gyroscopic.

## Hydroplaning effect

Figs 17,18,19,20 and 21 shows the effect of the tire width , track angle and wheel load on the critical speed of the vehicle due to hydroplaning At the angle of the road surface 2.29 degrees, it is noticed that the effect of increasing the tire width and thickness of the water layer at critical speed is negative while the effect of the wheel load has a positive effect on the at critical speed. . These figures show full agreement with those obtained by: Mets [6] And Muhsin et al. [7].



**Fig. 17.** The relation between Critical Speed and tire width for different wheel load.

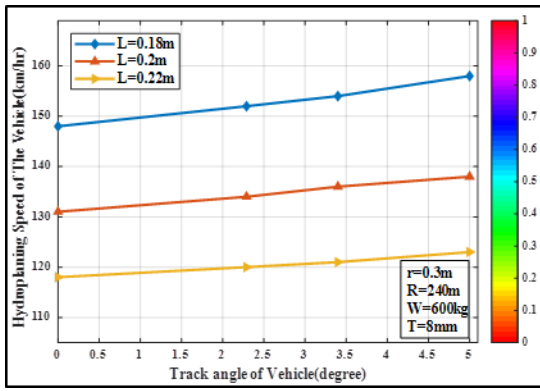


Fig. 18. The relation between Critical Speed and track angle for different tire width.

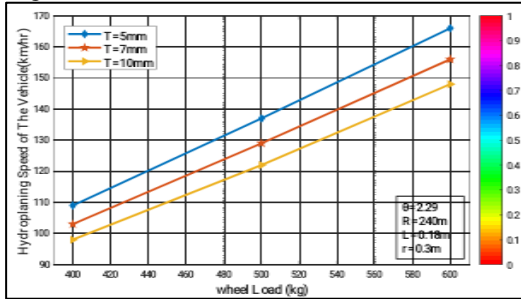


Fig. 19. The relation between Critical Speed and wheel load for different water layer depth.

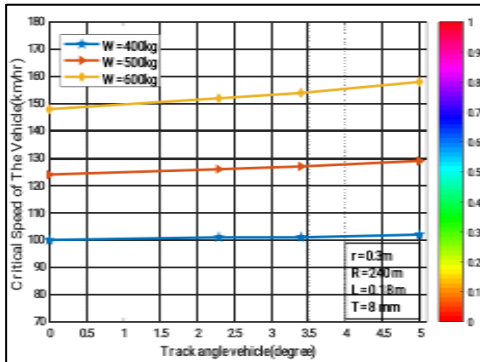


Fig. 20. The relation between Critical Speed and track angle vehicle for different wheel load.

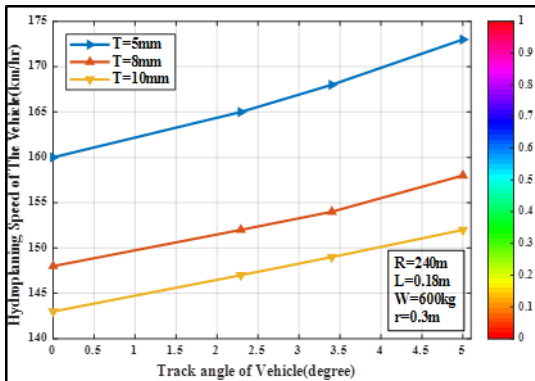


Fig. 21. The relation between Critical Speed and track angle vehicle for different water layer depth.

## Conclusions

- 1-The results showed that the effect of the gyroscopic couple negatively effects the safe speed of the vehicle
- 2-The relationship between the vehicle load, track angle of the vehicle, radius of the circular path and the vehicle width is direct in both cases; considering the effect of the gyroscopic torque or without it.
- 3-The relationship between the of the gravity center height the vehicle , Tire width the and the critical speed of the vehicle is inversely related.
- 4-The critical speed regarding hydroplaning at a horizontal cornering (track angle of the vehicle = zero) is exactly the same as for straight roads.

## References

- [1] Mahadevan S. 'Developing a Vehicle Hydroplaning Simulation using Abaqus and CarSim . PhD Thesis, Diss. Virginia Tech; 2016.
- [2] Martynenko YG, Merkurjev I, Podalkov V. Control of nonlinear vibrations of vibrating ring microgyroscope. *Mechanics of Solids* 2008; 43(3): 379-390.
- [3] Brach RM. An analytical assessment of the critical speed formula. SAE Paper No. 970957; 1997.
- [4] Chang T-H. Effect of vehicles' suspension on highway horizontal curve design. *Journal of transportation engineering* 2001; 127(1): 89-91.
- [5] Lambert K. A study of vehicle properties that influence rollover and their effect on electronic stability controllers. Master's thesis, Auburn university , 2007.
- [6] Metz LD. Hydroplaning Behavior during Steady-State Cornering Maneuvers. *SAE International Journal of Materials and Manufacturing* 2011;4(1):1068-1079.
- [7] Ibrahim A. Muhsin A. A Study of the effect of the wheel load, tire dimensions and water depth on the vehicle critical speed in wet roads. *Tikrit Journal of Engineering Sciences* 2013;20(2):1-7.
- [8] ŽurAulis V, Sokolovskij E, Matijošius J. The opportunities for establishing the critical speed of the vehicle on research in its lateral dynamics. *Eksplotacija i Niezawodność* 2013; 15 (4): 312–318
- [9] Khot A, Kumbhojkar N. Modeling and Validation of Prototype of Self Stabilizing

Two Wheeler Using Gyroscope. [singhad college of engineering, pure, Maharashtra, India](#) 2014; 5(12) 48-54.

- [10] Colvin G. Development and validation of control moment gyroscopic stabilization .Ph.D. dissertation, The Ohio State University, Columbus, OH, 2014.
- [11] Luo W, Wang KC, Li L. Hydroplaning on Sloping Pavements Based on Inertial Measurement Unit (IMU) and 1mm 3D Laser Imaging Data. [Periodica Polytechnica Transportation Engineering](#) 2016; 44(1): 42-49..
- [12] Mashadi B, Mokhtari-Alehashem M, Mostaghimi H. Active vehicle rollover control using a gyroscopic device. Proceedings of the Institution of Mechanical Engineers, Part D: [Journal of Automobile Engineering](#) 2016; 230(14): 1958-1971.
- [13] Gopinath A, Rooby AT, Babu S. Design and Fabrication of Self Balancing Two-Wheeler Model Using Gyroscope. [International Journal of Engineering, Technology, Science and Research](#) 2017; 9(3); 2051-2058.
- [14] Cameron, A. and Ettles, C. M. Mc. Basic Lubrication Theory", 3rd Edition, publ. Ellis Horwood, Chichester, UK, 1981.

M	Mass vehicle	Kg
$M_{Gx}$	the gyroscopic torques about g1	N.m
$M_{Gy}$	the gyroscopic torques about g2	N.m
$M_{Gz}$	the gyroscopic torques about g3	N.m
P	Water pressure trapped between tyre and road	Pa
r	tyre radius at the outer surface	M
V	Critical vehicle speed	km/hr
W	Load on one Wheel	Kg
$W_v$	Vehicle load	Kg
$\omega$	The angular velocity of the rotor axis	$\frac{rad}{s}$
$\omega_p$	The angular velocity of the rotary axis (precession)	$\frac{rad}{s}$
$\beta$	Half central corner of the underwater part of the tyre	degree
$\theta$	Angle of slope track	Degree

## NOMENCLATURES

Symbol	Description	Unit
a	vehicle Width	m
b	The area of the element on the outside of the tire	m <sup>2</sup>
f	Coefficient of lateral friction	-
$F_N$	The sum of the vertical forces on the surface of the road at the outer surface of the tire	N
$F_T$	The sum of horizontal forces on the surface of the road at the outer surface of the tire	N
$F_{up}$	The total vertical forces on the surface	N
g1	the precession axis	—
g2	the spin axis	—
g3	the torque axis	—
H	The thickness of the water slide in the rib area	m
$H_{C.G}$	Height of the center of gravity of the vehicle	m
$I_R$	the mass moment of inertia of the rotor around the spin axis	Kg. m <sup>2</sup>
L	width the tire	m