Evaluating the (q-d) Axis Parameters of Three Phase Salient Pole Synchronous Machine

Omar T. Mahmoud
Technical Institute, Haweja, Kirkuk, Iraq

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ABSTRACT

In order to perform any simulated work like using MATLAB Simulink for a salient pole synchronous machine the direct-axis (d-axis) synchronous reactance \( X_d \) and quadrature-axis (q-axis) synchronous reactance \( X_q \) and other parameters must be calculated. In this paper many tests has been made practically to calculate these parameters with the aid of using slip test and sudden short circuit test using data acquisition card. These parameters have been calculated successfully for the used salient pole machine.

Keywords
Synchronous machine parameters, MATLAB Simulink.

List of Symbols

\( X_d \) - Direct axis synchronous reactance (\( \Omega \)).
\( X_q \) - Quadrature axis synchronous reactance (\( \Omega \)).
\( I_d \) - Direct component of armature current (amp).
\( I_q \) - Armature current quadrature component (amp).
\( E_a \) - Internal induced voltage (volt).
\( X_d^t \) - Transient direct axis reactances (\( \Omega \)).
\( X_q^t \) - Subtransient quadrature axis reactances (\( \Omega \)).
\( \tau_d^t \) - d-axis short circuit transient time constant(s).
\( \tau_d^s \) - d-axis short circuit subtransient time constant(s).
\( E_{\text{max}} \) - Maximum armature voltage (volt).
\( E_{\text{min}} \) - Minimum armature voltage (volt).
\( I_{\text{min}} \) - Minimum armature current (amp).
\( I_{\text{max}} \) - Maximum armature current (amp).

* Corresponding author: E-mail address: omar_taaee2002@yahoo.com
Theoretical Concepts

Synchronous machine

Synchronous machine is one in which alternating current flows in the armature winding, and dc excitation is supplied to the field winding. The armature winding is almost invariably on the stator and is usually a three-phase winding, the field winding is on the rotor. The amplitude of the generated voltage is proportional to the frequency and the field current. The current and power factor is then determined by the machine field excitation and the impedance of the machine and loads. Per phase equivalent circuit of the synchronous machine is shown in Fig. (1). [1]

![Fig.(1) per Phase Equivalent Circuit of Synchronous machine](image)

Saliency Effect of Synchronous Machine

The synchronous machine is subdivided in to two types. The first type is when the rotor has a cylindrical construction where the air gap between the rotor and the stator is uniform which is called cylindrical rotor machine. On the other hand, a salient-pole rotor has a larger air gap in the region between the poles than in the region just above the poles. This difference is shown in Fig. (2) [2].

![Fig.(2) Salient-pole (left) and non-salient-pole (right)](image)

two components, the component along the pole axis is commonly called the direct axis (d-axis) synchronous reactance \(X_d\), and the other component along the axis between the poles (q-axis) is called the quadrature-axis synchronous reactance \(X_q\). In the same manner the armature current is also resolved into two components, \(I_d\) and \(I_q\). The direct component \(I_d\) produces a field lags the internal voltage \(E_a\) by 90°. The quadrature component \(I_q\) produces a field in phase with \(E_a\). Salient pole synchronous machine has many (d-q) parameters that must be evaluated in order to simulate this machine. Beside \(X_d\) and \(X_q\), there are other parameters these are:

1. Transient, subtransient direct axis reactances \(X_{d}'\) and \(X_{d}''\).
2. Transient, subtransient quadrature axis reactances \(X_{q}'\) and \(X_{q}''\).
3. Direct axis short circuit transient time constant \(\tau_d'\).
4. Direct axis short circuit subtransient time constant \(\tau_d''\). [2]

Tests of synchronous machine

Before performing any simulated tests to the machine, the machine parameters must be estimated. The machine type of our test rig is a salient pole machine, so that, direct-axis (d-axis) synchronous reactance \(X_{d}\) and quadrature-axis (q-axis) synchronous reactance \(X_{q}\) must be evaluated, this is done by performing the slip test to the machine. Also the transient,
subtransient reactances $X'_{d}$ and $X'_{q}$, and the d-axis time constants, direct axis short circuit transient time constant $\tau_{d}^{*}$, and direct axis short circuit subtransient time constant $\tau_{d}^{**}$, must be evaluated by a sudden short circuit test. To make the calculations easier and more general the quantities that are estimated for the machine have been converted to per unit system [3].

**Slip Test**

This test is conducted by driving the machine as a motor at a speed very slightly different from synchronous speed with the field open circuit and the armature energized by a three-phase, rated-frequency, and positive-sequence power source. The armature current, and the voltage are observed. The maximum and minimum values for the voltages and currents are measured by suitable ammeters. The minimum and maximum ratios of the armature voltage ($E_{\text{max}}$ and $E_{\text{min}}$) to the armature current ($I_{\text{min}}$ and $I_{\text{max}}$) are obtained when the slip is very small. From these, approximate values of quadrature-axis and direct-axis synchronous reactances $X_{d}$ and $X_{q}$ can be obtained using equations (1) and (2) [4].

\[
X_{d} = \frac{E_{\text{max}}}{I_{\text{max}}} \quad \text{(1)}
\]
\[
X_{q} = \frac{E_{\text{min}}}{I_{\text{min}}} \quad \text{(2)}
\]

**Sudden Short Circuit Test**

This test is an important one. It is used to evaluate the transient and subtransient reactances $X'_{d}$ and $X'_{q}$. From this test the values of $\tau_{d}$ and $\tau_{d}^{**}$ can be estimated too. These parameters are important to simulate the synchronous machine in the MATLAB Simulink program, sim power system, machines tool box in synchronous machines per unit standard type. The d-axis transient reactance $X'_{d}$ determined from the current waves after a three-phase short-circuits suddenly applied to the machine operating open-circuited at rated speed. $X'_{d}$ is equal to the ratio of the open-circuit voltage at the instant of the short circuit to the value of the armature current obtained by extrapolation of the envelope of the ac component of armature current wave to the instant of application of the short circuit (the transient envelop), neglecting the rapid variation of current during the first few cycles, this is clear in Fig.(3). The direct-axis subtransient reactance $X'_{d}$ is similarly determined by the same three-phase suddenly applied short-circuit test as used for determination of the transient reactance. It is equal to the R.M.S. value of the open circuit voltage at the moment of the short circuit divided by the R.M.S. value of the subtransient envelop of the current.

![Fig. (3) Oscillogram of three-phase sudden short circuit][3]

The direct axis short circuit transient time constant $\tau_{d}$, and the direct axis short circuit subtransient time constant $\tau_{d}^{*}$, are evaluated from the semi logarithmic plot of the difference between the transient envelop and the steady state amplitude (curve B) in Fig.(4). Similarly the difference between the subtransient envelop and the extrapolation of the transient envelop is plotted (curve A) in the same figure. Both plots closely approximate straight lines (C and D). The value of $\tau_{d}$ is equal to the time required for the transient envelop to decay to the point where the difference between this envelop and steady state amplitude equals to 0.368 of the initial difference. The value of $\tau_{d}^{*}$ equal to the time required for the subtransient envelop to decay to the point where the difference between this envelop and transient
envelop is equal to 0.368 of the initial difference as shown in the Fig. (4) [4].

Two formulas (3 and 4) have been used for this purpose as below.

\[
X_q = \frac{\text{rms value of prefault open circuit phase voltage}}{\text{initial value of the transient current} \times \sqrt{2}} \quad ... (3)
\]

\[
X_q = \frac{\text{rms value of prefault open circuit phase voltage}}{\text{initial value of the subtransient current} \times \sqrt{2}} \quad ... (4)
\]

The factor \(\sqrt{2}\) has been appeared here because the transient and subtransient envelops represent the peak current values.

The remaining parameters of the salient pole machine \((X_{q}, X_{d})\) are evaluated using table (1), (all the parameters values are in per unit).

In this paper, the extraction of three phase short circuit current waveform is achieved by an interface circuit between the machine output current and NI-PCI-6023E data acquisition card (DAQ) made by National Instrumentation Company. The benefit of using the (DAQ) card is to obtain an accurate drawing for the current waveform in the personal computer monitor. The interface circuit consists of a current transformers (CT) with a 375/7.5 transformation ratio (as it labeled in its data sheet), connected to each phase of the machine output current. To plot the machine current using the data acquisition card at 1500 sample/second and 750 msec/division, some hardware and software installation must be provided.

These installations needed for this purpose are:

1. MATLAB 7.0.
2. Data Acquisition tool box version 2.5.
3. Supported data acquisition software.

The current signals of the (CT) are connected to an accessory card, and then via a special wiring cable they are transferred to DAQ card and finally to the monitor of the personal computer. Finally Fig.5 shows the experimental test rig that is used to perform the above tests [5].

**Practical results and calculations**

Before any calculation has been made, the rating of the synchronous machine must be calculated in order to evaluate the parameters in per unit quantity. The synchronous machine ratings for the test rig that used in this paper are as follow:

**Synchronous Machine Ratings**

- Machine power: 1kW.
- Maximum DC excitation voltage: 110 volt.
- Maximum DC excitation current: 0.75A.
- Machine weight 370 Newton.
- Machine moment of inertia (J): 0.015 kg.m².
- Machine friction coefficient (K): 0.0035 kg.m²/sec.
- Machine power rated factor: 0.95
Machine rated speed and frequency:
1500 rpm, 50 Hz. [7].

And the machine base values are:
Machine base volt ampere \( (V\text{A}_b) \) = 1kVA.
Machine base voltage \( V_b = 380 \) volt phase to phase.

Machine base current
\[
I_b = \frac{V\text{A}_b}{\sqrt{3}V_b} = \frac{1 \times 10^3}{\sqrt{3} \times 380} = 1.5\text{A} \quad \ldots (5)
\]

Machine base impedance
\[
Z_b = \frac{V_b^2}{V\text{A}_b} = \frac{380^2}{1000} = 144.4\Omega \quad \ldots (6)
\]

**Results for synchronous machine slip test**

The below table (2) shows the results for the slip test of the synchronous machine. Using equations (1) and (2), the d-axis \((X_d)\) and q-axis \((X_q)\) reactances are:

- \(X_d\) in per unit value = 1.16\pu
- \(X_q\) in per unit value = 0.58\pu

**Results of a Sudden Short Circuit Test**

The machine phase current waveform which has been obtained from a sudden short circuit test with the aid of NI-PCI-6023E data acquisition card (DAQ). The current waveforms are obtained from a current transformer (CT) of a \((375/7.5)\) as a transformation ratio. Sudden short circuit current waveforms is shown in Fig.(6).

![Fig.6. Machine phase current waveform when a sudden short circuit is applied](image)

Transient and subtransient envelopes are drawn to obtain the current difference \(\Delta_i^t\) which is the current difference between the transient envelop and steady state current value, and \(\Delta_i^s\) which is the current difference between the transient and subtransient envelopes. The x-axis is the sample rate or the time of the short circuit current, its scale is 1500 sample/second and 750msec/division. From Fig.(6) it is easy to calculate the d-axis transient and d-axis subtransient reactances \(X_d^t\) and \(X_d^s\) as it is clear from Fig.(6) the transient initial current is equal to \((3.4745\text{amp})\), and the subtransient initial current is equal to \((5.6\text{amp})\) and using equations (4 and 5) so that

\[
X_d^t = \frac{380/\sqrt{3}}{3.4745/\sqrt{3}} = 09.54\Omega \quad \text{In per unit value}
\]

\[
X_d^s = \frac{380/\sqrt{3}}{5.6/\sqrt{3}} = 55.55\Omega \quad \text{In per unit value}
\]

The final results of \(\Delta_i^t\) and \(\Delta_i^s\) are listed in table (3). The first three columns of this table represent the three current values of the machine after it have been multiplied with the transformation ratio of the CT, which are the steady state current \(i\), transient current \(i^t\) and subtransient current \(i^s\). The next two columns represent the current differences \(\Delta_i^t\) and \(\Delta_i^s\). The final column represents the time in sample per second. These values have been calculated by plotting the steady state transient, and subtransient envelops manually on Fig.(6) as in the Fig.(7).

![Fig.7. Envelops for machine sudden short test](image)

The two current differences \(\Delta_i^t\) and \(\Delta_i^s\) are drawn with time in a semi logarithmic
paper as in Figs. (8) and (9). The results are approximately two straight lines, the first is in Fig.(8), used to find $\tau_d$, and the second one is in Fig.(9) is used to find the value of $\tau'_d$. The time scale of these two figures is 1500 sample/second and 750 msec/division respectively.

The value of $\tau_d$ is defined as the time required for the transient envelop to decay to the point where the difference between this envelop and the steady state envelop is equal to 0.368 of the initial difference. While the value of $\tau'_d$ is defined as the time required for the subtransient envelop to decay to the point where the difference between this envelop and the transient envelop is 0.368 of the initial difference. Depending on the two previous definitions and from Figs. (8) and (9) the following results are obtained:

$$\tau_d = 0.04 \text{ sec} \quad \text{and} \quad \tau'_d = 0.028 \text{ sec}$$

Discussion

The results in this work have a small approximation in order to evaluate a suitable parameter and more accurate simulation for the salient pole synchronous machine, this approximation has a little effect on the whole simulation of the machine because we made many tests like open and short circuit tests for the practical and simulated machine and we have approximately the same results.

References

Table. 1 Typical values of synchronous machine constants[2]

<table>
<thead>
<tr>
<th>Machine constants</th>
<th>$X_d$(pu)</th>
<th>$X'_d$(pu)</th>
<th>$X''_d$(pu)</th>
<th>$X_q$(pu)</th>
<th>$X'_q$(pu)</th>
<th>$X''_q$(pu)</th>
<th>$T_d$(pu)</th>
<th>$T'_d$(pu)</th>
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</thead>
<tbody>
<tr>
<td>Salient pole generators</td>
<td>1.00</td>
<td>0.35</td>
<td>0.23</td>
<td>0.65</td>
<td>0.65</td>
<td>0.65</td>
<td>1.8</td>
<td>0.035</td>
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Table. 2 Slip test results

<table>
<thead>
<tr>
<th>Current and voltage</th>
<th>Step one</th>
<th>Step two</th>
<th>Step three</th>
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<tbody>
<tr>
<td>$i_{max}$ (A)</td>
<td>0.58</td>
<td>0.515</td>
<td>1.41</td>
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<tr>
<td>$i_{min}$ (A)</td>
<td>0.53</td>
<td>0.452</td>
<td>0.77</td>
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<tr>
<td>$V_{max}$ (volt)</td>
<td>90</td>
<td>78</td>
<td>130</td>
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<tr>
<td>$V_{min}$ (volt)</td>
<td>88</td>
<td>76</td>
<td>118</td>
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</table>

Table. 3 Results for a sudden short circuit test applied to synchronous machine

<table>
<thead>
<tr>
<th>$i$ (amp)</th>
<th>$i'_i$ (amp)</th>
<th>$i''_i$ (amp)</th>
<th>$\Delta i'_i$ (amp)</th>
<th>$\Delta i''_i$ (amp)</th>
<th>Time in Sample per second</th>
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<tr>
<td>1.462</td>
<td>3.4745</td>
<td>5.6</td>
<td>2.0125</td>
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<td>3300</td>
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