

## Effect of Tooth Geometry on Gear Pump Performance

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### Abstract

In this paper, the capability to use unstandard gears and their effect on the performance of the gear pumps are investigated. These effects include pump flow rate, trapped volume between two meshing gears, and pulsation flow rate factor.

Equations for the pump flow rate are derived and a computer program has been written to evaluate all parameters and performance. The gear used in this work is unstandard one with correction factor of (+ 0.5).

The simulation results showed that the theoretical flow rate increases when unstandard gears are used. Results showed also that the trapped volume and pulsation flow rate factor are reduced when unstandard gears are used.

**Keywords:** Gear pump, Pump performance, Pulsation flow rate.

### تأثير شكل السن على أداء المضخة الترسية

#### الخلاصة

تم في هذا البحث دراسة امكانية استخدام التروس غير القياسية في المضخات الترسية وتأثيرها على أداء المضخة من حيث معدل التدفق ومقدار حجم المائع المحصور بين ترسين معشقين ومعامل التدفق النبضي، واستنتاج المعادلات الخاصة بأداء المضخة الترسية بتروس غير قياسية، بمعامل تصحيح مقداره (+0.5).

وعند مقارنة النتائج المستحصلة تم التوصل الى ان أداء المضخة بتروس غير قياسية افضل من ادائها بتروس قياسية، حيث اثبتت النتائج ان معدل التدفق النظري يزداد عند استخدام تروس غير قياسية، كذلك فان حجم المائع المحصور بين ترسين معشقين ومعامل التدفق النبضي يقل عند استخدام التروس غير القياسية.

**الكلمات الدالة:** المضخة الترسية، أداء المضخة، معدل التدفق النبضي.

#### Notations

$A_o$  : theoretical distance between centers of meshing gears (mm).

$b$  : width gear (mm).

$i$  : gear ratio.

$m$  : module (mm).

$n$  : gear pump rotation (r.p.m).

$\Delta p$  : pressure difference at intake and discharge end of the gear pump ( $N/m^2$ ).

$R$  : distance between center of gear and contact point (mm).

$R_{a1}$  : distance between center of drive gear and contact point of addendum circle (mm).

$R_{a2}$  : distance between center of driven gear and contact point of addendum circle (mm).

$R_{p1}$  : distance between center of drive gear and contact point of pitch circle (mm).

$R_{p2}$  : distance between center of driven gear and contact point of pitch circle (mm).

$t$  : circular pitch (mm).

$X$  : final value of correction factor.

$X_1$  : correction factor of drive gear.

$X_2$  : correction factor of driven gear.

$Z$  : number of teeth.

#### Greek letters

$\alpha$  : pressure angle of standard gear (Deg.).

$\alpha_o$  : pressure angle of unstandard gear (Deg.).

$\delta$  : pulsation flow rate factor.

$\varepsilon$  : contact ratio of standard gear.

$\varepsilon_o$  : contact ratio of unstandard gear.

## Introduction

An external straight teeth spur gear pump is a constant displacement pump with a circular motion. In outward appearance, it resembles a centrifugal pump, but it differs from a centrifugal pump in action. While it continuously scoops the liquid out of the pump chamber, the latter only imparts a velocity to the stream of fluid. It combines the constant discharge characteristic of the centrifugal type with positive discharge features of the reciprocating one. It is simpler in construction, has much smaller dimensions for a given capacity, occupies less space and costs less to install and maintain<sup>[1]</sup>.

The basic geometry and nomenclature of a spur gear mesh is shown in Figure (1).

Modern fluid control system requires design of gear pumps with predictable features and operational characteristics. Hence, it is of utmost importance to derive mathematical expressions for ideal delivery, pulsation flow rate, etc; and to analyze the design parameters influencing the pump characteristics to understand their effect so as to aid the process of

improved design of such pumps to be incorporated in industrial applications for dependable and satisfactory performance<sup>[2]</sup>.

## Survey of Previous Research

Ichikawa T., Yamaguchi and Willekens<sup>[3]</sup> in 1971 tried to develop analytical model for ideal flow and flow pulsation of an external spur gear pump. Yanda H. A.<sup>[4]</sup> in 1987 described the measurement of the pressure in the trapped space. The resulting signals being taken out through pressure taps Ali K H.<sup>[5]</sup> in 1989 investigated and measured the pressure distribution

around the gear rotor using miniature pressure transducer positioned at gear fillet. Marius G.<sup>[6]</sup> in 2000 studied the behavior of the journal bearing and its effect on side plate of external gear pump. Noah D., and Suresh B. Kasaragadda<sup>[7]</sup>, in 2003 studied the theoretical flow ripple of an external gear pump using different numbers of teeth on driving and driven gear. Ngamura K., Ikejo K., and Tututan F. G., in 2004<sup>[8]</sup> investigated the design and performance of external gear pumps with a tooth profile based on the cycloid tooth profile. Hyun Kim, Hazel Marie, and Suresh patil<sup>[9]</sup>, in 2007 used computation fluid dynamic to analyze the two dimensional flow of external gear pump.

## Modeling

Following are the specifications of the standard and unstandard external gear pumps that are selected for this study.

### A) Standard Gear:

Module : 4.5 (mm)  
 Width of gear : 36 (mm)  
 Correction factor : 0  
 Pressure angle : 20 (Deg.)  
 Speed : 3000 (r.p.m)  
 Number of Teeth : 8 – 10 – 12 – 14 – 16

### B) Unstandard Gear:

Module : 4.5 (mm)  
 Width of gear : 36 (mm)  
 Correction factor : 0.5  
 Pressure angle : 31.3167 (Deg.)  
 Speed : 3000 (r.p.m)  
 Number of Teeth : 8 – 10 – 12 - 14 – 16

## Theory

### *Gears Properties Unstandard*

The new gears technology uses the unstandard gears in most application for the following reasons<sup>[10,11]</sup>:

- 1.To avoid under cutting phenomena when teeth used were less than 17.

2.To increase the resistance of the tooth root and fillet region against the stress exposed on the root.

3.To reduce the contact ratio between meshing teeth.

Positive correction factor ( $X_o$ ) leads to increase the tooth thickness at pitch circle and to decrease it at tooth tip, so that the tooth shape changes according to the value of the used correction factor as shown in Figures (2) and (3). Also the following coefficients will take the following formulas<sup>[10,11]</sup>:

1. Diameter of the addendum circle:

$$D_a = 2(A + m - X_o m) - (D_p)_g \dots\dots(1)$$

2. Root circle diameter of tooth:

$$D_r = (D_p)_g - 2(1.25 - X_o)m \dots\dots(2)$$

3. Pitch meshing circle diameter of gear:

$$(D_p)_e = mZ + 2X_o m \dots\dots(3)$$

4. Distance between centers of meshing gears:

$$A = A_o + Xm \dots\dots(4)$$

$$A_o = m \frac{Z_1 + Z_2}{2} \dots\dots(5)$$

$$X = X_1 + X_2 = 2X_o \dots\dots(6)$$

5. Pressure angle:

$$\alpha = \cos^{-1}(\cos(\alpha_o) \frac{A_o}{A}) \dots\dots(7)$$

6. Base circle diameter:

$$D_b = (D_p)_g \cos(\alpha_o) \dots\dots(8)$$

7. Circular pitch:

$$t = m\pi \dots\dots(9)$$

8. Diameter of the generation pitch circle:

$$(D_p)_g = mZ \dots\dots(10)$$

**Ideal Delivery of a Gear Pump**

An analytical expression for the theoretical flow rate of a gear pump can be deduced by energy method which involves the concept of an ideal torque. This ideal torque is produced by the pressure difference of fluid at intake and discharge end of the gear pump<sup>[2]</sup>.

Referring to Fig.(4), the ideal torque of the driving gear can be written as:

$$T_1 = b \int_{R_1}^{R_{a1}} \Delta P.R.dR = \Delta P b / 2(R_{a1}^2 - R_1^2) \dots\dots(11)$$

And that of the driven gear can be expressed as:

$$T_2 = b \int_{R_2}^{R_{a2}} \Delta P.R.dR = \Delta P b / 2(R_{a2}^2 - R_2^2) \dots\dots(12)$$

Let the driving gear rotate through an angle  $d\theta_1$  and the corresponding rotation of the driven one be  $d\theta_2$ ; then the equation of energy can be written as:

$$T_1 d\theta_1 + T_2 d\theta_2 = \Delta P \partial q \dots\dots(13)$$

$$\partial q / \partial \theta_1 = (T_1 + T_2.i) / \Delta P \dots\dots(14)$$

Where:

$$i = d\theta_2 / d\theta_1 = Z_2 / Z_1$$

Theoretical flow rate for each rotation of gear is given by the following formula:

$$Q = \pi b [R_{a1}^2 - R_{p1}^2 + i(R_{a2}^2 - R_{p2}^2) - (1-i)t_o^2 / 12] \dots\dots(15)$$

$$t_o = \pi m \cos \alpha_o \dots\dots(16)$$

**Standard Gears**

For standard gears properties<sup>[5]</sup>:

$$\left. \begin{aligned} R_{a1} &= R_{p1} + m \\ R_{a1}^2 - R_{p1}^2 &= 2R_{p1}m + m^2 \\ R_{a2}^2 - R_{p2}^2 &= 2R_{p2}m + m^2 \end{aligned} \right\} \dots\dots(17)$$

For gear pump having two identical involute profile external spur gears,  $D_{a1} = D_{a2} = D_a$ ,  $D_{p1} = D_{p2} = (D_p)_g = mZ$ ,  $Z_1 = Z_2 = Z$ ,  $i = Z_1 / Z_2 = 1$ .

Hence equation (15) for ideal flow delivery is modified to:

$$Q_T = 2\pi b m^2 n [Z + (1 - \frac{\pi^2 \cos^2 \alpha_o}{12})] * 10^{-6} \dots\dots(18)$$

**Unstandard Gears**

For unstandard gear pump having two identical involute profile external spur gears,

$$D_{a1} = D_{a2} = D_a, D_{p1} = D_{p2} = (D_p)_e, Z_1 = Z_2 = Z, i = \frac{Z_1}{Z_2} = 1, D_{p1} = D_{p2} = (D_p)_g, X_1 = X_2 = 2 X_o. \text{ Hence equation (15) for ideal flow delivery is modified to:}$$

$$(QT)_c = n\pi b \{ (mZ + mX + m - mX_o - \frac{mZ}{2})^2 - [\frac{m(Z + 2X_o)}{2}]^2 - \frac{m^2 \pi^2 \cos^2 \alpha}{6} \} * 10^{-6} \dots\dots\dots(19)$$

**Contact Ratio**

**Contact Ratio of Standard Gears**

Contact ratio of standard gears can be calculated by the following formula<sup>[12]</sup>:

$$\epsilon = \frac{2\sqrt{R_a^2 - R_b^2} - A \sin \alpha_o}{t_o} \dots\dots(20)$$

$$t_o = \pi m \cos \alpha_o \dots\dots\dots(21)$$

**Contact Ratio of Unstandard Gears**

For unstandard gears pump apply the following formulas:

$$R_a = (mZ + mX + m - mX_o - \frac{mZ}{2})^2 \dots\dots(22)$$

$$\alpha = \cos^{-1} \cos \alpha_o \frac{A_o}{A} \dots\dots\dots(23)$$

$$A = A_o + Xm = mZ + Xm \dots\dots\dots(24)$$

$$R_b = mZ \cos \alpha_o \dots\dots\dots(25)$$

$$t_o = m\pi \cos \alpha_o \dots\dots\dots(26)$$

Substituting equations 22-26 in equation 20 will give the following formula :

**Pulsation Flow Rate Factor**

**For Standard Gear pump**

For standard gear pump having two identical involute profiles, backlash between the teeth, and trapped liquid volume. The pulsation flow factor has the following expression<sup>[12,13]</sup>.

$$\delta_2 = \frac{\pi^2 \cos^2 \alpha_o}{4[Z + 1 - \frac{\pi^2 \cos^2 \alpha_o}{12}]} \dots\dots(28)$$

**For Unstandard Gear pump**

For unstandard gear pump having two identical involute profiles, backlash between the teeth, and trapped liquid volume. The pulsation flow factor in equation (28) has the following expression:

$$\delta_4 = \frac{\pi^2 \cos^2 \alpha_o}{4[(mZ + mX + m - mX_o - \frac{mZ}{2})^2 - \frac{\pi^2 \cos^2 \alpha_o}{12} - (m(Z + 2X_o)/2)^2 - \frac{\pi^2 \cos^2 \alpha_o}{12}]} \dots\dots(29)$$

**Results and Discussions**

**Ideal Flow Rate**

The ideal flow rate is one of the important properties of gear pump. Therefore, in this paper the possibility of increasing the flow rate has been studied. Figure (5) shows that the flow rate is proportional with the number of teeth. Figure (6) shows that the flow rate is proportional with square of the module of the gears. Figure (7) shows that the flow rate is proportional with the width of teeth. Figure (8) shows that the flow rate is proportional with the pressure angle of gears. The results shown in the above figures give an increase in flow rate when using unstandard gear with correction factor of (0.5) instead of standard gear. To get high flow rate with small size of pump it is perfect to increase the module and decrease number of teeth. Also the pressure angle increases when the number of teeth decreases, as shown in Figure (9), and this increases the flow rate.

**Trapped Volume**

In the gear pump when meshing ends between pair contact teeth an enclosed volume (Trapped Volume) between

contact points is created, for the other pair of meshing teeth. During gears rotation the enclosed volume will decrease causing an increase in pressure, because of the trapped liquid inside enclosed space<sup>[4]</sup>. Therefore, a gear with a few contact ratios is used. Figure (10) shows that contact ratio is proportional with number of teeth. Figure (11) represent the relation between trapped volume and contact ratio of standard gear. Figure (12) represents the relation between trapped volume and contact ratio of unstandard gear. Because of the decrease in contact ratio an unstandard gears are used as shown in Figure (12). Therefore unstandard gears are used to decrease the trapped liquid volume.

#### ***Pulsation Flow Rate Factor***

Pulsation flow rate produced from gear pump is in the form of pulses. These phenomena in gear pump leads to vibrations inside the pump and affect the efficiency of the pump<sup>[3]</sup>. Figure (13) shows that the pulsation flow rate factor decreases as the number of teeth increases for unstandard gear pump. As this factor decreases the efficiency of the pump increases. Comparison with Figure (14), the results show that the unstandard gear is better than standard gear.

#### **Conclusions**

1. The results proved that the unstandard gear pumps give a high flow rate in comparison with standard gear pumps.
2. The trapped liquid volume decreases and also the contact ratio between teeth meshing decreases. This is produced when an unstandard gear is used.
3. The pulsation flow rate factor decreases with an increase in the number of teeth. However, an increase in the number of teeth leads to stabilization in pulsation flow rate.

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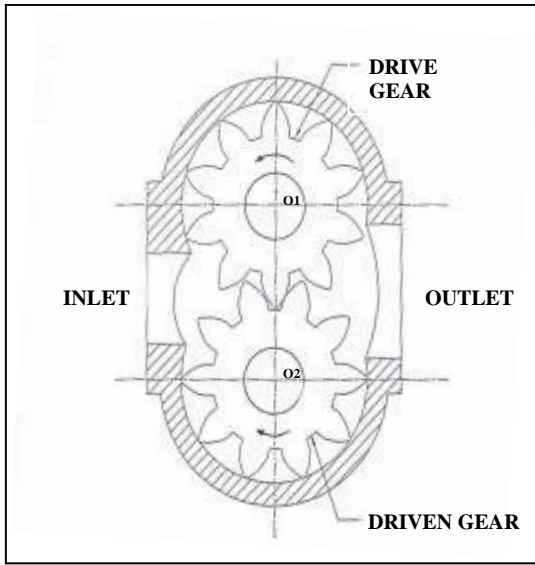


Fig. (1): Diagram of external meshing of gear pump

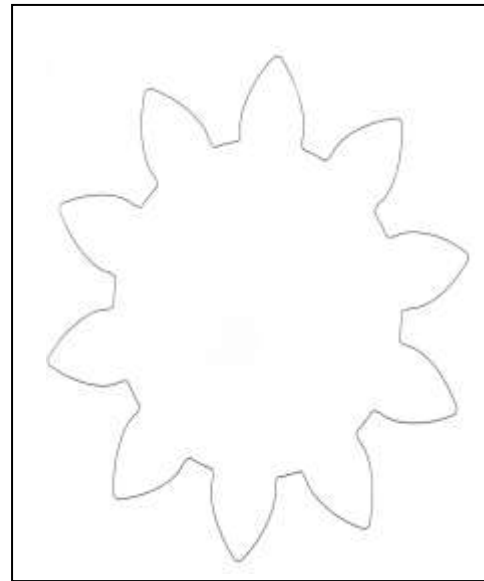


Fig. (3): Unstandard gear

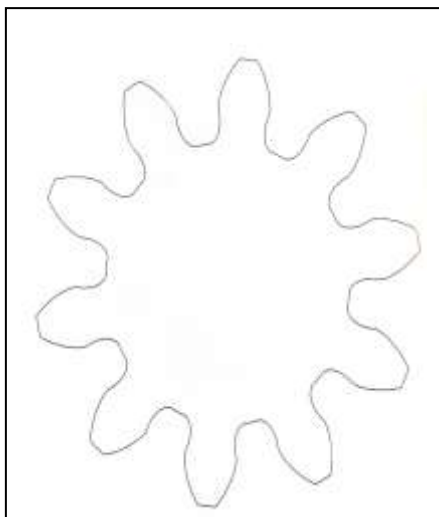


Fig. (2): Standard gear

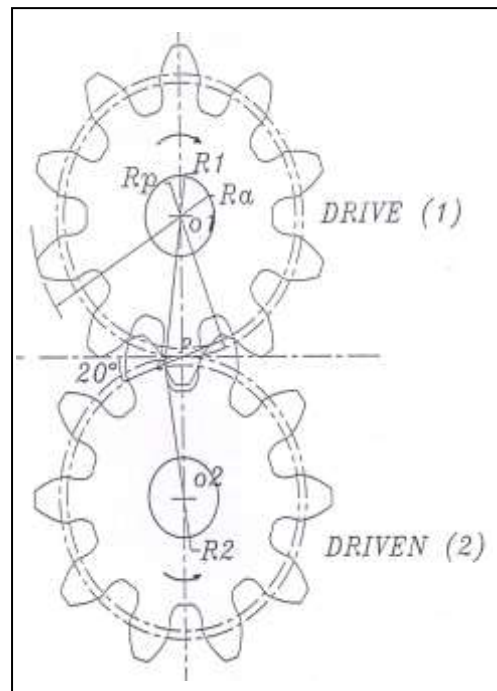


Fig. (4): Gear pump mesh flow rate.

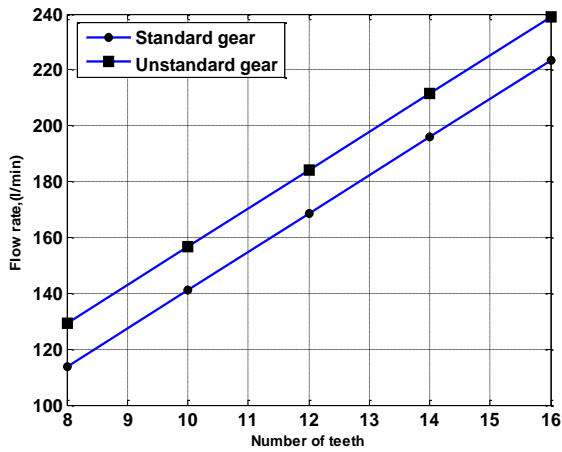


Fig. (5): Relation between number of teeth and theoretical flow rate

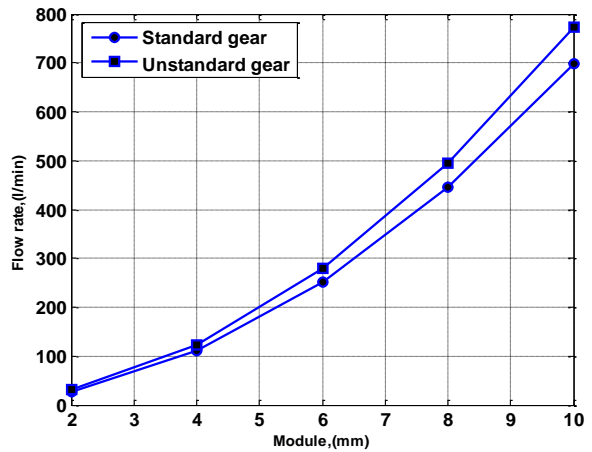


Fig.(6): Relation between module and flow rate

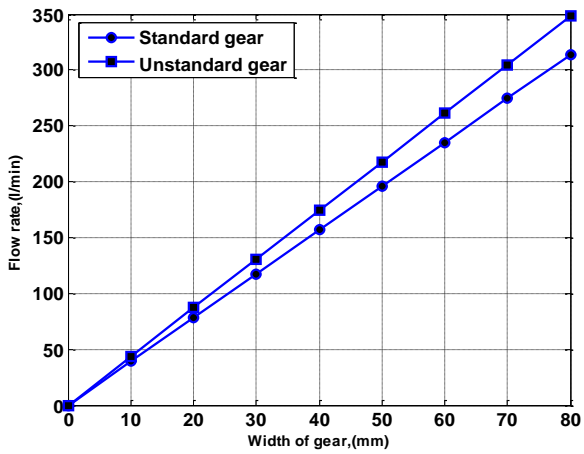


Fig. (7): Relation between width of gear and flow rate

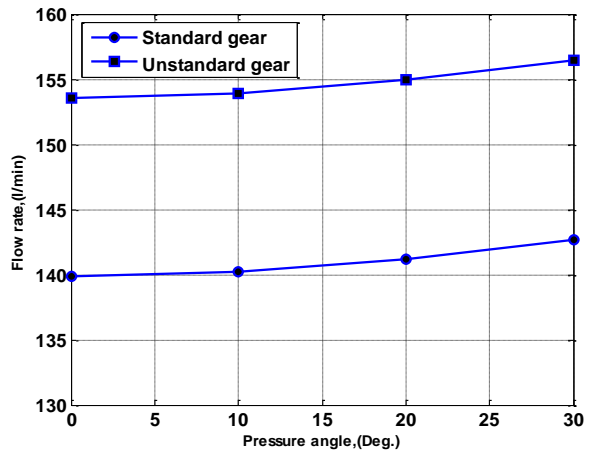


Fig.(8):Relation between pressure angle and flow rate

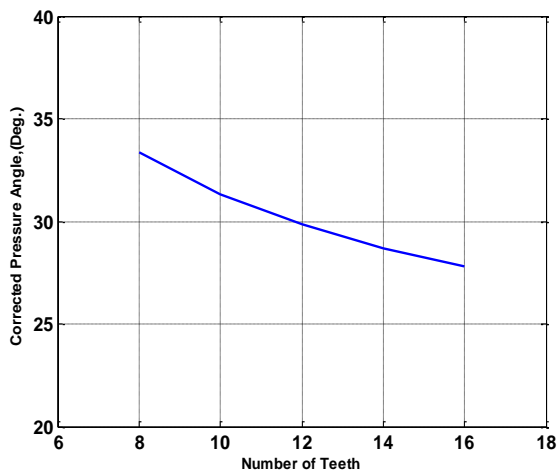


Fig. (9): Relation between number of teeth and pressure angle of unstandard gear

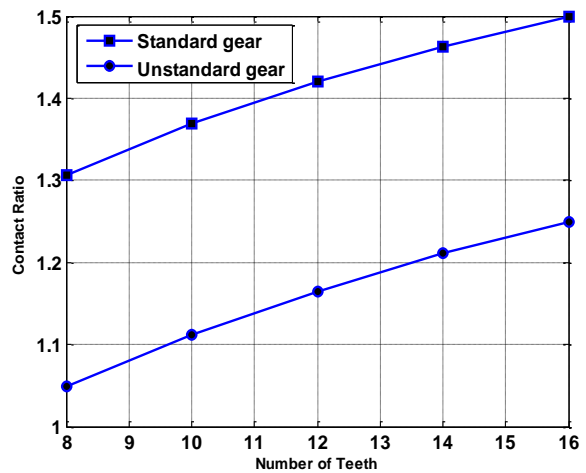
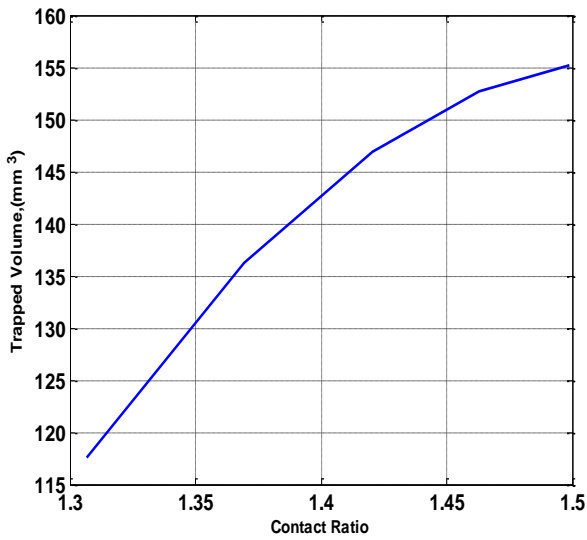
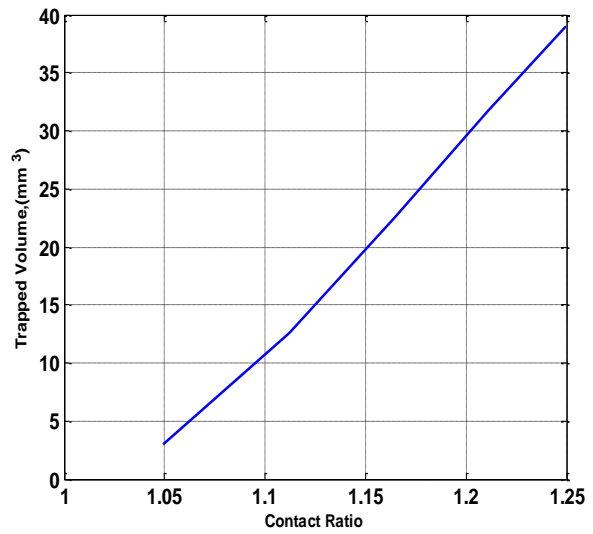


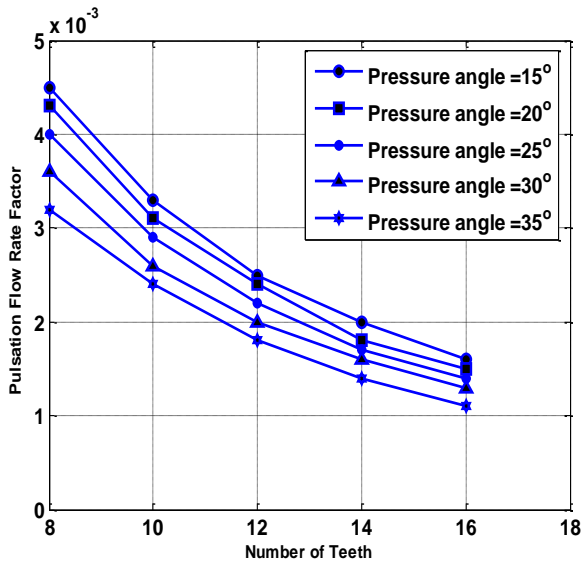
Fig. (10): Relation between number of teeth and contact ratio



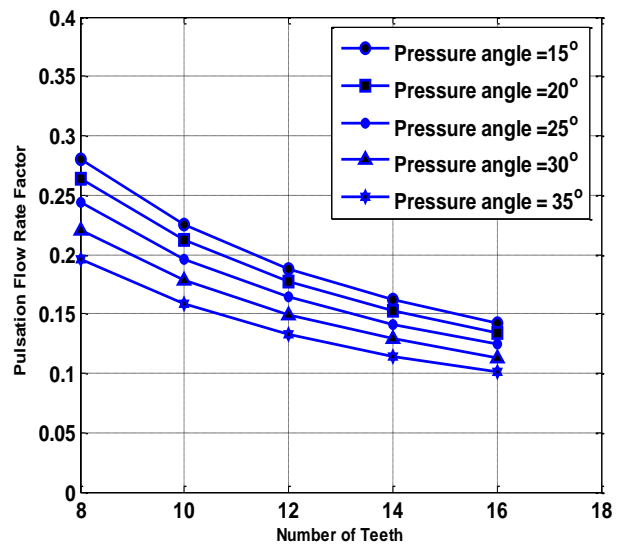
**Fig. (11): Relation between contact ratio and trapped volume for standard gear**



**Fig.(12): Relation between contact ratio and trapped volume for unstandard gear**



**Fig. (13): Relation between pulsation flow rate factor and number of teeth of unstandard gear**



**Fig. (14): Relation between pulsation flow rate factor and number of teeth of standard gear**