Pressure - Settlement Characteristics of Shallow Foundations using Finite Element Method

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(Received 24 January 2016, Accepted 19 April 2016, Available online 31 March 2017)

Abstract

The present study attempts to represent the behavior of shallow foundations under the effect of eccentric inclined loading in terms of the main criteria of design which are the ultimate bearing capacity, permissible settlement, horizontal displacement and tilt. Due to the square shape of the analysed foundations, three dimensional finite element analysis are used. Elastic-perfectly plastic behavior of soil and rigid of foundations was adopted using Mohr-Coulomb criterion, 15-Node wedge elements were used to model and represent the soil and 5-Node linear elements with three degree of freedom to model and represent the foundations in used program PLAXIS 3D TUNNEL version 1.2. The results of analysis were presented in the form of pressure-settlement, pressure-horizontal displacement and pressure-tilt characteristics. Then the ultimate bearing capacity of the foundations were gotten and compared with (Meyerhof, 1956) and (Saran & Agrawal, 1991), a good agreement was found between them. Using the data obtained from the analysis, non-dimensional correlations have been developed for predicting the values of settlement, horizontal displacement and tilt of eccentrically-obliquely loaded foundations. These relationships can be used by the engineers.

Keywords: Shallow foundation, sand, eccentric-inclined load, non-dimensional correlations.
**Introduction**

Square footing is the most common type of shallow foundations that is usually used to distribute individual column loads to the surrounding soil.

In general, these foundations are subjected to vertical load, horizontal load and a moment, the resultant of these becomes eccentric inclined load on the foundation, Figure (1).

In Figure (1), the reference position and current position of the footing are shown. The symbols used are:
- \( S_c \) = settlement
- \( h \) = horizontal displacement
- \( \theta \) = tilt

**Fig. 1.** Shallow Foundation Subjecte to eccentric-inclined load  
(after Butterfield et al. 1997)

Engineers are often required to evaluate the behavior of shallow foundations subjected to such loading and this is especially true in the problems of off-shore structures, retaining walls, columns and portal frames. Most of the published experimental data and analytical models relate to the two dimensional planar version of the problem of the shallow foundations under eccentric inclined loading conditions (Ameen, 2008)[1].

Also, there are studies about analytical three dimensional shallow foundation but it is few. Meyerhof (1953)[2] was first to study the behavior of shallow foundations under eccentric inclined loading by experimental model. The concept of reduced width for analyzing the eccentrically loaded footings was developed.

Meyerhof (1956)[3] suggested an empirical relation to compute the ultimate bearing capacity of footings subjected to eccentric-inclined loads.

Vesic (1973)[4] proposed a reduction factor to be applied in the bearing capacity equation when footing is subjected to eccentric inclined load.

Agrawal (1986)[5] performed strip, square and rectangular footings model tests on dry sand to study the behavior of footings under eccentric-inclined loads.

Nova and Montrasio (1991)[6] suggested a method to evaluate settlement and rotations of rigid shallow foundations on sand under the combined action of inclined and eccentric loads.

Ngo-Tran (1996)[7] used the FEM (finite element method) to examine the elastic behavior and stability of circular footings under combined loads using two dimensional axi-symmetric analysis.

Al-Samadi (1998)[8] studied the behavior of ring footings subjected to eccentric inclined load resting on dry sand (Dr = 70%) and he found that, for the same pressure intensity, tilt decreases with an increase in the size of footing.

Bouzid, et al (2005)[9] presented a new approximate method called the Vertical Slices Model (VSM) based on a combination of 2D finite element and finite difference methods. The method was used to predict the behavior of an embedded square footing under combined loading in a non-homogeneous half-space where the stiffness profile was modeled as a power-low of depth.

Saleh et al (2008)[10] used a laboratory work and numerical analysis to study the behavior of one sided skirted strip footing subjected to eccentric inclined load, they found that the increasing length of the skirted improve the load – settlement behavior.
Ameen (2008)[1] investigated the behavior of rectangular footing on c-ϕ soil and concluded equations to estimate the vertical settlement and tilt of rectangular footing subjected to eccentric inclined load.

Al-Azzawi (2010)[11] investigated the behavior of rectangular and square footing resting on gypseous soil.

**Problem Definition**

In this paper, an investigation of the behavior of square footings resting on dry pure sand subjected to eccentric inclined loads with many parametric study was done, Table (1). In this study, the commercial finite element program PLAXIS 3D TUNNEL version 1.2 was used.

**Table 1.** Parametric study

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Range of values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth to width ratio D/B</td>
<td>0.0, 0.5, 1.0</td>
</tr>
<tr>
<td>Eccentricity to width ratio e/B</td>
<td>0.0, 0.05, 0.1, 0.15</td>
</tr>
<tr>
<td>Load inclination angle with respect to the vertical, i (degree)</td>
<td>0, 4, 8, 12</td>
</tr>
<tr>
<td>Relative density of sand Dr (%)</td>
<td>84, 46, 9.5</td>
</tr>
<tr>
<td>Dimensions of footing (mm)</td>
<td>500<em>500</em>300</td>
</tr>
<tr>
<td></td>
<td>750<em>750</em>450</td>
</tr>
<tr>
<td></td>
<td>1000<em>1000</em>600</td>
</tr>
</tbody>
</table>

**Materials and Methods**

**Model development**

In this study, the soil was modeled using three dimensional element with 5-Node wedge and six stress points (stress or Gauss point), Figure (2).

The footing was modeled using linear element with five nodes with three degree of freedom $(u, v, \dot{\vartheta})$.

Figure (3) shows a finite element mesh that used capacity of footing in this study. The lateral and bottom boundaries of the finite element meshed were change according to the width of footing (Bowels, 1988)[12].

The dimensional in the x-direction = $B + 4B*2$

The dimensional in the y-direction = $B + 4B*2$

The dimensional in the z-direction = $6B$

Where $B$ is the footing width.

The footing was considered to be rigid and rough, as it most often is in reality and was modeled as elastic with much greater stiffness than the soil (footing stiffness $(E) = 26*10^6$ kN/m², unit weight of concrete $(\gamma) = 24$ kN/m³, Poisson’s ratio of concrete $(\mu) = 0.2$)(Bowles, 1996)[13]. The soil was modeled with Mohr coulomb yield criterion and assumed that the soil is elastic perfectly plastic material, it's properties recorded in Table (2).

![Fig. 2. Three dimensional element with 15-Node wedge and 6 stress points](image-url)
Table 2. Properties of the used soil (from Agrawal, 1986)

<table>
<thead>
<tr>
<th>Soil properties</th>
<th>Soil type</th>
<th>Loose Sand</th>
<th>Medium Sand</th>
<th>Dense Sand</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulus of elasticity, E kN/m²</td>
<td></td>
<td>37500</td>
<td>27500</td>
<td>23500</td>
</tr>
<tr>
<td>Unit weight, γ kN/m³</td>
<td></td>
<td>16.3</td>
<td>15.2</td>
<td>14.3</td>
</tr>
<tr>
<td>Poisson’s ratio, µ</td>
<td></td>
<td>0.26</td>
<td>0.29</td>
<td>0.34</td>
</tr>
<tr>
<td>Internal friction, φ degree</td>
<td></td>
<td>41</td>
<td>36</td>
<td>29.5</td>
</tr>
<tr>
<td>Cohesion, c kN/m²</td>
<td></td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>Angle of dilatancy, ψ degree</td>
<td></td>
<td>11</td>
<td>6</td>
<td>0.0</td>
</tr>
<tr>
<td>Interface reduction factor, R_{intr.}</td>
<td></td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Model Verification
A comparison was made between the used program and an earlier study (Agrawal, 1986)[5]. The comparison gave a good agreement. Figure (4) shows some curves.

Results and Discussion
Curves were drawn between the settlement (S_e), horizontal displacement (δ_h) and tilt (t) with the applied pressure. From pressure-settlement curves, bearing capacity values were found (the bearing capacity was found by De Beer method at D_f = 0 and by Tangent method at D_f = 0.5, 1.0, Figures (4) and (5), and compared with (Meyerhof 1956[3], Saran & Agrawal 1991[14]). Table (3) shows the bearing capacity of footing in this study, and Figures (7) shows the
comparison of bearing capacity with another studies.

Also, values of settlement (maximum settlement, Sm and settlement at point load, Se) and horizontal displacement were found, Table (4). These values were divided by So (So is a vertical settlement under central vertical load) to get a non-dimensional values, Table (5).

![Fig. 4a](image1.png)

**Fig. 4a.** Comparison of settlement between Present study and Agrawal 1986 for \( (e/B = 0 \text{ and } i=0^\circ) \)

![Fig. 4b](image2.png)

**Fig. 4b.** Comparison of settlement between Present study and Agrawal 1986 for \( (e/B = 0 \text{ and } i=5^\circ) \)

![Fig. 4c](image3.png)

**Fig. 4c.** Comparison of settlement between Present study and Agrawal 1986 for \( (e/B = 0 \text{ and } i=10^\circ) \)
Fig. 4d. Comparison of settlement between Present study and Agrawal 1986 for (e/B = 0 and i=15°)

Fig. 4e. Comparison of settlement between Present study and Agrawal 1986 for (e/B = 0 and i=20°)

Fig. 5. Non dimensional pressure-settlement curve of square footing on dense sand (De Beer, 1970)
Table 3. Comparison of bearing capacity on dense sand

<table>
<thead>
<tr>
<th>No.</th>
<th>e/B</th>
<th>i°</th>
<th>Ultimate Bearing Capacity (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>D/f/B = 0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>B (mm) 500</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>769.9</td>
</tr>
<tr>
<td>2</td>
<td>0.05</td>
<td>4</td>
<td>636.7</td>
</tr>
<tr>
<td>3</td>
<td>0.05</td>
<td>8</td>
<td>542.7</td>
</tr>
<tr>
<td>4</td>
<td>0.05</td>
<td>12</td>
<td>458</td>
</tr>
<tr>
<td>5</td>
<td>0.1</td>
<td>0</td>
<td>719.9</td>
</tr>
<tr>
<td>6</td>
<td>0.1</td>
<td>4</td>
<td>586.8</td>
</tr>
<tr>
<td>7</td>
<td>0.1</td>
<td>8</td>
<td>488</td>
</tr>
<tr>
<td>8</td>
<td>0.1</td>
<td>12</td>
<td>360</td>
</tr>
<tr>
<td>9</td>
<td>0.15</td>
<td>0</td>
<td>552.8</td>
</tr>
<tr>
<td>10</td>
<td>0.15</td>
<td>4</td>
<td>487.7</td>
</tr>
<tr>
<td>11</td>
<td>0.15</td>
<td>8</td>
<td>404.9</td>
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<tr>
<td>12</td>
<td>0.15</td>
<td>12</td>
<td>327.6</td>
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<tr>
<td>13</td>
<td>0.15</td>
<td>0</td>
<td>517.7</td>
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<tr>
<td>14</td>
<td>0.15</td>
<td>4</td>
<td>480</td>
</tr>
<tr>
<td>15</td>
<td>0.15</td>
<td>8</td>
<td>401</td>
</tr>
<tr>
<td>16</td>
<td>0.15</td>
<td>12</td>
<td>315.8</td>
</tr>
</tbody>
</table>

Fig. 6. Tangent method to find the value of bearing capacity of square footing on dense sand (D/f = 0.5, B = 1000, e/B = 0.1, i = 8°)
Table 4. Se, Sm and δh for square footing on dense sand (B = 1000mm)

<table>
<thead>
<tr>
<th>No</th>
<th>Df/B</th>
<th>ε/B</th>
<th>p</th>
<th>Factor of safety = 1</th>
<th>Factor of safety = 2</th>
<th>Factor of safety = 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Se (mm)</td>
<td>Sm (mm)</td>
<td>δh (mm)</td>
</tr>
<tr>
<td>1</td>
<td>0.15</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>6</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>0.05</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>0.05</td>
<td>0</td>
<td>0</td>
<td>4</td>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>0.05</td>
<td>0</td>
<td>0</td>
<td>5</td>
<td>6</td>
<td>12</td>
</tr>
</tbody>
</table>
Table 5. $\text{Se}/\text{So}$, $\text{Sm}/\text{So}$ and $\delta h/B$ for square footing on dense sand 
(B=1000mm)

<table>
<thead>
<tr>
<th>No.</th>
<th>$D/B$</th>
<th>$e/B$</th>
<th>$p$</th>
<th>Factor of safety = 1</th>
<th>Factor of safety = 2</th>
<th>Factor of safety = 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\text{Se}/\text{So}$</td>
<td>$\text{Sm}/\text{So}$</td>
<td>$\delta h/B$</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0 1 1 0 1 0 1 1 0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>4 0.86 0.87 0.383 0.833 0.833 0.004</td>
<td>0.292 0.292 0.004</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>4 0.86 0.87 0.383 0.833 0.833 0.004</td>
<td>0.292 0.292 0.004</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>4 0.86 0.87 0.383 0.833 0.833 0.004</td>
<td>0.292 0.292 0.004</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.15</td>
<td>0.15</td>
<td>0.15</td>
<td>4 0.86 0.87 0.383 0.833 0.833 0.004</td>
<td>0.292 0.292 0.004</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>4 0.86 0.87 0.383 0.833 0.833 0.004</td>
<td>0.292 0.292 0.004</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>4 0.86 0.87 0.383 0.833 0.833 0.004</td>
<td>0.292 0.292 0.004</td>
<td></td>
</tr>
</tbody>
</table>

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Fig. 7. Comparison of bearing capacity on dense sand

**Non – Dimensional Correlations-Settlement (Se)**

**For dense sand**

\[
\frac{S_e}{S_o} = (-0.047 i^3 + 0.947 i^2 - 4.742 i + 23.86) \\
\left(\frac{\epsilon}{B}\right)^2 + (0.012 i^3 - 0.237 i^2 + 1.07 i - 5.116) \left(\frac{\epsilon}{B}\right) \\
- 0.021 i + 0.979 ........................................(0)
\]

**For medium dense sand**

\[
\frac{S_e}{S_o} = (-0.1645 i^3 + 3.1543 i^2 - 14.745 i + 30.02) \left(\frac{\epsilon}{B}\right)^2 \\
+ (0.0209 i^3 - 0.4186 i^2 + 2.1059 i - 5.949) \left(\frac{\epsilon}{B}\right) \\
- 0.03 i + 0.9817 ..............................................(2)
\]

**For loose sand**

\[
\frac{S_e}{S_o} = (0.0393 i^3 - 0.8396 i^2 + 4.6414 i + 23.46) \left(\frac{\epsilon}{B}\right)^2 \\
+ (0.002 i^3 - 0.0228 i^2 + 0.0181 i - 5.285) \left(\frac{\epsilon}{B}\right) \\
- 0.0276 i + 0.9956 ..............................................(3)
\]

**Maximum Settlement (Sm)**

**For dense sand**

\[
\frac{S_m}{S_o} = (0.0315 i^3 - 0.5851 i^2 + 5.9934 i + 14.269) \left(\frac{\epsilon}{B}\right)^2 \\
+ (-0.0014 i^3 + 0.029 i^2 - 0.5996 i - 2.37) \left(\frac{\epsilon}{B}\right) \\
- 0.0284 i + 1.0036 ..............................................(4)
\]

**For medium dense sand**

\[
\frac{S_m}{S_o} = (-0.0543 i^3 + 0.899 i^2 - 1.999 i + 40.309) \left(\frac{\epsilon}{B}\right)^2 \\
+ (0.0106 i^3 - 0.1779 i^2 + 0.5711 i - 6.1939) \left(\frac{\epsilon}{B}\right) \\
- 0.0318 i + 1.0183 ..............................................(5)
\]

**For loose sand**

\[
\frac{S_m}{S_o} = (-0.1936 i^3 + 3.4064 i^2 - 11.808 i + 44.793) \left(\frac{\epsilon}{B}\right)^2 \\
+ (0.0282 i^3 - 0.5142 i^2 + 2.0136 i - 6.8931) \left(\frac{\epsilon}{B}\right) \\
- 0.0289 i + 0.9958 ..............................................(6)
\]
Horizontal Displacement ($\delta_h$)

For all types of sand

$$\frac{\delta_h}{B} = \left(0.5079i^2 - 10.581i + 55.459\right)\left(\frac{i}{\phi}\right)^2$$

$$+ (-0.1028i^2 + 2.1848i - 12.82)\left(\frac{i}{\phi}\right)$$

$$- 0.0026i^2 + 0.0574i - 0.0165$$

Where:

$S_o$: vertical settlement under central vertical load.
$S_e$: settlement at point load.
$S_m$: maximum settlement.
$\delta_h$: horizontal displacement.
e: eccentricity.
B: footing width.
i: load inclination.
$\phi$: friction angle.

Influence of the Footing’s Width

A non-dimensional relations were drawn between $q/B\gamma$ and $Se/B$ to study the influence of width of the footing (B), Figure (8). It was found that increasing the width caused increasing the settlement under the same pressure.

Pressure Settlement Relationship

From pressure – settlement curves, it was noticed that increasing the applied load's inclination caused increasing in horizontal displacement. While increasing in eccentricity to width ratio or footing embedment to width ratio caused decreasing in horizontal displacement.

Pressure Tilt Relationship

The value of tilt was calculated from the equation below:

$$(8)$$

$$\sin t = \frac{S_m - S_e}{B - e}$$

where :

$S_m$: maximum settlement of footing at the pressure applied.
$S_e$: settlement of footing at the point of applied pressure.
B: width of footing.
e: eccentricity.
t: tilt of footing.

From which curves were noticed that the tilt of footing increase by increasing the load inclination ($i$) and the eccentricity to width ratio ($e/B$), and decrease by increasing the footing embedment to width ratio ($D/B$).

$0.098 \quad 57.76687$

<table>
<thead>
<tr>
<th>q/B\gamma</th>
<th>se/B</th>
<th>q/B\gamma</th>
<th>se/B</th>
<th>q/B\gamma</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>6.134969</td>
<td>0.002417</td>
<td>5.816769</td>
<td>0.000794</td>
<td>2.944785</td>
</tr>
<tr>
<td>12.26994</td>
<td>0.005201</td>
<td>11.63354</td>
<td>0.00164</td>
<td>5.889571</td>
</tr>
<tr>
<td>18.40491</td>
<td>0.008777</td>
<td>17.45031</td>
<td>0.002638</td>
<td>8.834356</td>
</tr>
</tbody>
</table>

Fig. 8a. Influence of B at $e/B = 0$ and $i=0^\circ$
Fig. 8b. Influence of $B$ at $e/B = 0$ and $i=4^\circ$

Fig. 8c. Influence of $B$ at $e/B = 0$ and $i=8^\circ$

Fig. 8c. Influence of $B$ at $e/B = 0$ and $i=12^\circ$
Conclusions

1- Analysis the square shallow foundation at different depth under vertical, inclined, eccentric, inclined-eccentric loads using PLAXIS 3D TUNNEL program gave a good agreement when compared with previous studies.

2- The ultimate bearing capacity decreases when the load inclination and eccentricity increase, the settlement of footing increases when the load inclination and eccentricity increase, the horizontal displacement increases when the load inclination increase while it decreases when the eccentricity increase. Using the embedment footing improve the bearing capacity of soil.

3- A non-dimensional correlations were gotten and can be used in engineering design. This correlation to predict the settlement, tilt and horizontal displacement

4- The correlations to predict the tilt and settlement are dependent upon eccentricity-width ratio, density of soil and inclination of load, and they are independent upon factor of safety, depth-width ratio, and width of footing.

5- The correlation to predict the horizontal displacement is dependent upon density of soil and inclination of load, and it is independent upon factor of safety, eccentricity-width ratio, and depth-width ratio.

References


