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A Comparative Study on the Performance Augmentation of a Gas Turbine Power Plant

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Abstract

In the present work, a comparative performance analysis of the gas turbine power plant with and without power augmenting devices was carried out. The intercooler, turbine burner (reheater), and heat exchanger were inserted individually and together to reveal the effect of each device. A model for each addition was derived on the base of the simple cycle and adapted to reflect the effect of the performance parameters on the plant power and thermal efficiency. The heat exchanger with different effectiveness improves the thermal efficiency but for low pressure ratios, while the intercooler has low effect at low pressure ratios. Its effect increases with increasing of this ratio and this needs advanced materials. The intercooler cooling percent was considered as 50%, 75% and 100%. The use of turbine burner alone augments the output power by 19.4% but on the expense of thermal efficiency (dropped by 4%) and fuel price. The interstage compressor cooling augments the power by 1.3%, while the efficiency increases slowly and needs a large amount of compression. The combination of intercooling with turbine burner enhances the power by 13.4%. So the regeneration must be installed with them to ensure an enhancement in plant compatibility and thermal efficiency.

Keywords: Gas turbine, Power plant, Performance enhancement, Intercooling, Regeneration, Reheater, Heat exchanger.

دراسة ادائية مقارنة عن فرص تعزيز اداء المحطة التوربينية الغازية

الخلاصة

في هذا البحث تم اجراء تحليل ادائي مقارنة عن المحطة التوربينية الغازية مع وبدون معززات القدرة. اضيف تبريد بين الضاغطة، الحارق التوربيني، والمبادل الحراري (اعادة التوليد) بشكل مفرد ومجمعة لبيان تأثير كل مكون. طور النموذج الرياضي للمحطة الغازية التقليدية بعد اجراء الاشتقاقات المطلوبة ليظهر اداء المحطة مع كل معزز ومن ثم اضافتهن مجتمعة لتعكس تأثير هذه المكونات على قدرة وكفاءة المحطة. ان المبادل الحراري بفاعلية مختلفة يحسن كفاءة المحطة لنسب انضغاط اطننة، لذلك فان تأثيره يزداد بزيادة نسبة الانضغاط ولكن ذلك يحتاج الى معدن مصنع بتقنية عالية. تم اعتماد نسب للتبريد بواقع 50%، 75% و 100%. ان استخدام الحارق التوربيني (اعادة التسخين) عزز القدرة بنسبة 19.4% ولكن على حساب الكفاءة (حيث انهارت بواقع 4%) و صرفيات الوقود. اعتماد التبريد الداخلي تسبب في زيادة القدرة بمعدل 1.3% و بزيادة قليلة في الكفاءة وحاجة اكبر الى زيادة في نسبة الانضغاط. جمع المعززات معا أدى الى زيادة الكفاءة بمقدار 13.4% وهذا يبين أهمية اضافة المبادل الحراري في حال استخدام التبريد الداخلي والحارق الخلفي معا.

الكلمات الدالة: التوربين الغازي، محطات قدرة، تعزيز الأداء، التبريد الداخلي، اعادة التوليد، اعادة التسخين، المبادل الحراري.

Nomenclature

C_P	Specific Heat at Constant Pressure kJ/kg.K
T	Temperature K
W	Work kW
x	Cooling Fraction
GT	Gas Turbine
TIT	Turbine Inlet
n	Net.

Subscripts

g	Gas
it	Inlet of the Turbine
t	Turbine
LPt	Low Pressure Turbine
mix	Mixture
c	Compressor
m	Mechanical
a	Air

Introduction

Performance enhancements by adding features, initially to the gas turbine part of the cycle or as supplementary to the existing plant, improve efficiency and increase power output. Some are effective during the whole performance envelope, while others are used for performance augmentation during periods of demand.

Gas turbine technology with fuel flexibility and comparatively lowest cost/kW will play an important role in coming decades in the power generation market. R. K. Bhargava*, M. Bianchi et al. [1].

R.W. Scheicher [2] examined the key factors affecting the Brayton cycle efficiency such as turbine inlet temperature, compressor and turbine efficiencies, recapture effectiveness and proposed near-term values for fusion power plant studies based on existing products.

Jijun Zhou, Dan Turner [3] investigated the overall performance of the plant operation under different conditions using EOP. This research shows how a well-designed commercial software is exploited in engineering research and developments.

The use and advantages of an engine performance using the developed simulation tools was demonstrated by K. Mathioudakis, A. Alexiano [4]. This software is adapted to simulate the steady state and transient performance of the plant. F. Liu and W. A.

Sirignano [5] performed a thermodynamic cycle analysis to compare the relative performance of the conventional engine and the turbine burner engine. Turbine burner engine are shown to provide significantly higher specific power with only small increase in specific fuel consumption. It also widens the operating range of the plant.

B. Sheikhbeigi and M. b. Ghofrani [6] investigated a performance improvement of gas turbine power plant using reheat and reheat/recuperator. The results obtained on the basis of the developed model show that reheating in the context of a realistic study may lead to an improvement both in efficiency and in specific network using recuperator cause to decrease NOx emission. The influence of air cooling on gas turbine performance and thermodynamic assessment of various advanced cycles was presented by R.K. Sullerey, and Ankur Agarwal [7]. Results showed that an increase of 5% in power and 0.38% in efficiency using inlet air cooling.

Chang D. K. and Suk C. Chung [8] developed a program for the steady state and transient performance of 200 kW engine with free power turbine. The results show the engine was at maximum power output and minimum specific fuel consumption at 100% rpm in both the gas generator and the power turbine.

Q.M. Jaber [9] presented the influence of air cooling intake on the gas turbine performance. A comparison between using different cooling systems, i.e., evaporative and cooling coil, is performed. A computer simulation model for the employed systems is developed in order to evaluate the performance of the studied gas turbine unit, at Marka Power Station.

Xiaojun Shi and Defu Che [10] presented a parametric analyses are performed for the proposed combined cycle to evaluate the effects of several factors, such as the gas turbine inlet temperature (TIT), the condenser pressure, the pinch point temperature difference of the condensing heat exchanger and the fuel gas heating temperature on the performance of the proposed combined cycle through simulation calculations.

Amir Abbas Zadpoor et al. [11] utilized both inlet air cooling and inter-cooling within the proposed system to limit the decrease of the air mass flow contained in the given

volume flow as well as reduce the compression power required.

The work of B.T. Aklilu and S.I. Gilani [12] aims to develop mathematical models to simulate a single shaft gas turbine based cogeneration plant with variable geometry compressor.

The open-cycle gas turbines offered low capital costs, compactness, and efficiency close to that of the steam plants. Nevertheless, after the oil crisis in the 1970s the efficiency of power plants became the top priority, and combined-cycle plants, first in the form of existing steam plant repowering, and later, as specially-designed gas-and-steam turbine plants, have become a common power plant configuration [13].

Efficiencies of the simple-cycle early gas turbines were practically increased by incorporating intercooling, regeneration (or recuperation), and reheating. The back work ratio of a gas-turbine cycle improves as a result of intercooling and reheating[14].

In the beginning, gas turbines were inefficient, bulky, and unreliable engines. In order to improve their performance, modifications, such as reheat, intercooling, or recuperation, were applied.

Gas turbine based power plants have been favored in recent time as a result of the changes described in the previous section. Compared to large power stations such as coal fired stations and nuclear stations, the capital investment of gas turbine driven power plants is lower and the construction lead times are shorter [15].

In the present work, the effect of certain performance and design constraints on the behavior of the plant was done. Some of them are; total compression ratio, turbine inlet temperature, heat exchanger effectiveness, compressor and turbine efficiencies, cooling fraction, etc. A cycle analysis is adapted to reflect the plant performance in terms of the abovementioned parameters. Many interesting envelopes and figures of merits were constructed.

Performance Considerations

Gas turbine based power plants have been favored in recent time as a result of the changes described in the previous section. In addition, the gas turbine based power plants

provide sufficient operational flexibility to adjust the power generation schedule based on the fast changing power demand and market electric price [16].

The basic gas turbine cycle has low thermal efficiency which decreases in the hard climatic conditions of operation, so it is important to look for improved gas turbine based cycles [17].

Intercooling and reheating will always decrease the thermal efficiency unless they are accompanied by regeneration. This is because intercooling decreases the average temperature at which heat is added, and reheating increases the average temperature at which heat is rejected. Therefore, in gas-turbine power plants, intercooling and reheating are always used in conjunction with regeneration. These improvements, of course, come at the expense of increased initial and operation costs, and they cannot be justified unless the decrease in fuel costs offsets the increase in other costs.

Performance Augmentation

Operational flexibility is one of the most important characteristics of the gas turbine based power plant, which is important for power plant operating in a dynamic environment. The output rate of the power plant can be adjusted to ensure the optimal dynamic market by manipulating its operating conditions, i.e. the load mode, fuel type and power augmentation.

Thermodynamic analysis can be a perfect tool for identifying the ways of improving the efficiency of fuel use, and determining the best configuration for a cogeneration plant [3].

The methodology of performance prediction of the present work is based on the cycle analysis of the simple gas turbine based power plant. The necessary derivation of the efficiency and power were done for the plant with and without augmenting devices.

The most interesting aspect of this study is the presentation of the capability of each augmenting device of improving the power and/or the plant thermal efficiency. The selection of enhancement method is a vital decision involving technical as well as economic considerations.

The complete mathematical model of the gas turbine power plant with and without power augmentation is presented in Appendix.

The first law of thermodynamics applies on the compressor based on cold air properties assumption, reduces to Equation (1):

Eliminating T1 Equation (2):

For isentropic compression and expansion relations Equation (3):

Applying in Equation (2) Equation (4):

In the same manner, the specific network for the turbine given as Equation (5):

Arranging the above equations, taking into consideration the definition of the mechanical efficiency, the net specific work can be written as Equation (6):

And the efficiency of the simple gas turbine plant is mathematically arranged to reflect the effect of pressure ratio on the performance of the GT Equation (7).

When the station numbering is per the sketch shown in Figure (1).

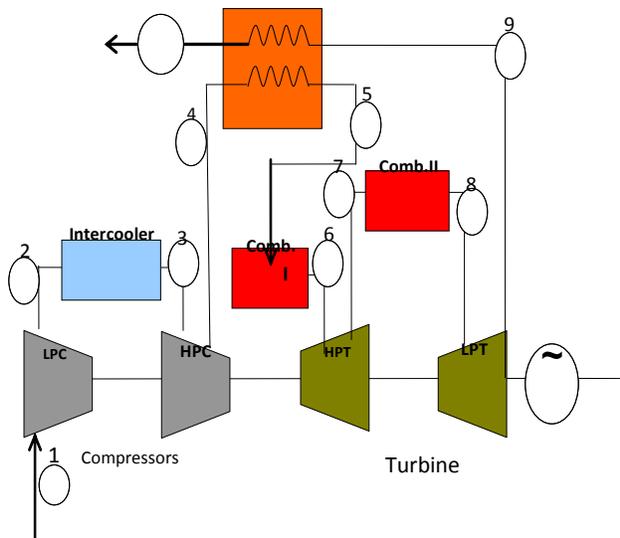


Fig. 1. Sketch of the gas turbine power plant with station numbering

Carrying out the compression process in stages and cooling the gas in between the lower and higher-pressure stages will decrease the work required to compress a gas between two specified pressures. This is called multistage compression with

intercooling. Intercooling may be used to reduce the work of compression between two given pressures in any application. The network is then can be given as Equation (8):

While the thermal efficiency of the inter-cooled gas turbine plant is derived. It included the flexibility of fractional cooling between compressors; thus Equation (9):

Where x is the cooling percentage (50%, 75% and 100%).

The exhaust gases of gas turbine power plant carry a significant amount of thermal energy that is usually expelled to the atmosphere. This causes a reduction in net work and efficiency of gas turbine. This problem can be solved by installing one of the available technologies of regeneration. The effectiveness of the heat exchanger, or regenerator, is a measure of how well it uses the available temperature potential to raise the temperature of the compressor discharge air. Specifically, it is the actual rate of heat transferred to the air divided by the maximum possible heat transfer rate that would exist if the heat exchanger had infinite heat transfer surface area. The equations of work and efficiency may then be given as Equation (10) & Equation (11).

The reheat cycle is used to extract more work in more than one expansion. Heat is added in the second combustor until the limiting temperature is reached. An intercooling is then required. The efficiency of the plant with turbine burner and intercooling is derived to be Equation (12).

The power output of the low pressure turbine can be increased by rising the temperature at inlet to the stage. This can be done by placing a second burner between the two turbines in order to heat the gases leaving the high pressure turbine. The addition of turbine burner increases the turbine work without changing the compressor work and this leads to an increase in the network. The efficiency may be given as Equation (13).

The combination of intercooling, reheat, and regeneration has the net effect of raising the average temperature of heat addition and lowering the average temperature of heat rejection. Insertion of the equations of each device in the equation of simple cycle leads to the following equation Equation (14).

Results and Discussion

A very important thing to be considered is that all the values (temperatures, efficiencies, pressure ratios, etc.) have been chosen theoretically but very close to the reality since all the power stations in Iraq don't have such values or information to give as the suppliers consider it as classified information, so it will be more applicable when this kind of information will be reachable and available.

A cycle performance methodology for the simple gas turbine power plant is carried out considering specified ranges of performance parameters. The efficiencies of compressors and turbines are considered 85% and 88% respectively. The polytropic efficiency is 0.9. The total pressure ratio is taken up to 30. The turbine inlet temperature is considered in the range of 1000 K to 1600 K. The performance analysis of other configurations that covers the installation of enhancing equipment's was then conducted. This methodology is adapted to reflect the influence of the key parameters and each equipment function upon the power generated and the efficiency of the GT plant.

Figure (2) shows the dependence of thermal efficiency on the compression ratio with regeneration at different effectiveness. In regeneration, the heat energy is recovered from the exhausted gases and used to preheat the combustion air.

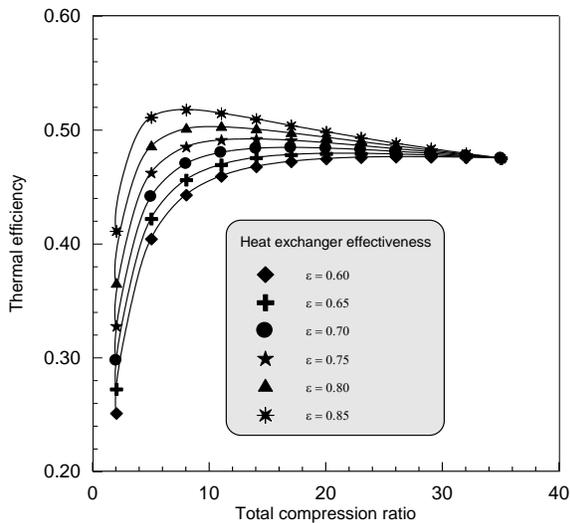


Fig. 2. Effect of Pressure Ratio and Heat Exchanger Effectiveness

The use of regeneration provides that the temperature of the turbine exit must be higher than that of the compressor discharge. This leads to the limitation that the use of heat exchanger is only powerful flow low range of total compression ratios. As the gases gain heat in the heat exchanger, then the gasses enter the burner with a relatively high temperature. This leads to low fuel required to attain the reference turbine inlet temperature and low fuel consumption especially with high heat exchanger effectiveness, Figure 3.

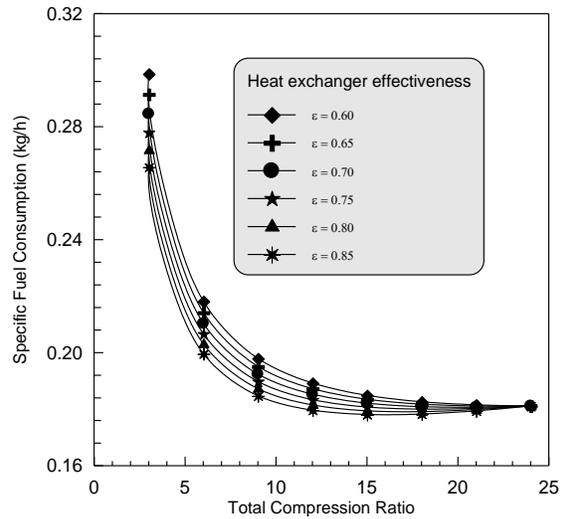


Fig. 3. Effect of Regeneration on Specific Fuel Consumption

The use of intercooling between compressors reduces the work required to drive the compressor which represents two-third of the power produced by the turbine. The thermal efficiency increases slowly and it need higher amount of compression, Figure 4. This will be faced with limitation of material durability. Furthermore, the use of intercooling is then reduces the fuel required to provide the work which derive the compressor. Figure 5 shows the dependence of specific fuel consumption on interstage compressor intercooling by 50% percent. Figure 6 reveals the power generated for the gas turbine power cycle with and without enhancements. The use of turbine burner compared with simple cycle for the same parameters augmented the power generated by 19% but the efficiency deteriorated approximately by 4% and the cost increases because of more fuel burnt, Figure

7. Therefore, the reheat is preferred in aero engines when access power is required for a short period. The use of regeneration is very powerful but unfortunately a limited range of total compression.

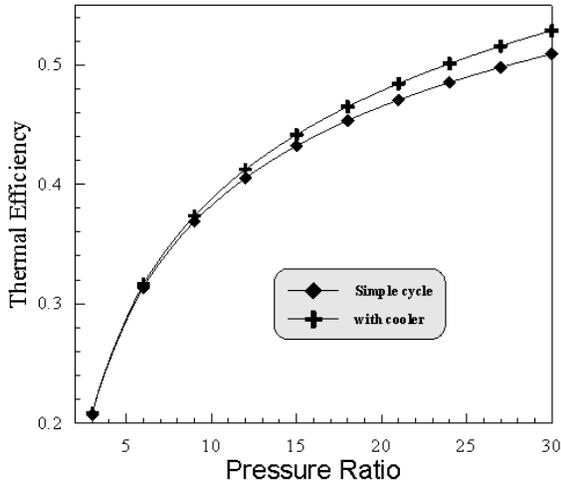


Fig. 4. Effect of Intercooling on Thermal Efficiency

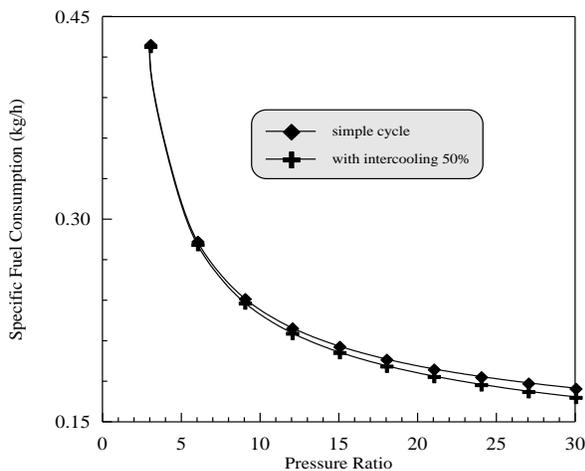


Fig. 5. Influence of Intercooling (50%) on Fuel Consumption

Results show that the use of performance enhancements devices (Intercooling, turbine burner and regeneration) improves the plant efficiency by 9%. The use of intercooling between compressors reduces the work required to derive the compressors. This permits to use more fuel to attain the reference turbine inlet temperature that does not change the turbine work. The use of

intercooling increased the efficiency by 3.4% for a total compression ratio of 15. It needs higher amount of compression. The fuel consumption of the plant with and without enhancement is presented in Figure 8. Thermal efficiency–power envelope presented in Figures 9 &10 which show that it is useful to achieve the maximum efficiency for the corresponding power. Figure 11 shows the effect of turbine inlet temperature on the plant efficiency. It holds good to get higher turbine inlet temperature but it will be limited with the restriction of materials and available technologies

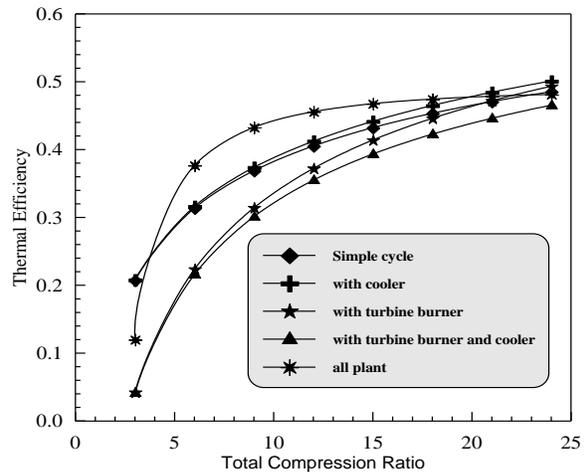


Fig. 7. Thermal Efficiency Dependence on Pressure Ratio for Augmenting Devices and Simple Cycle

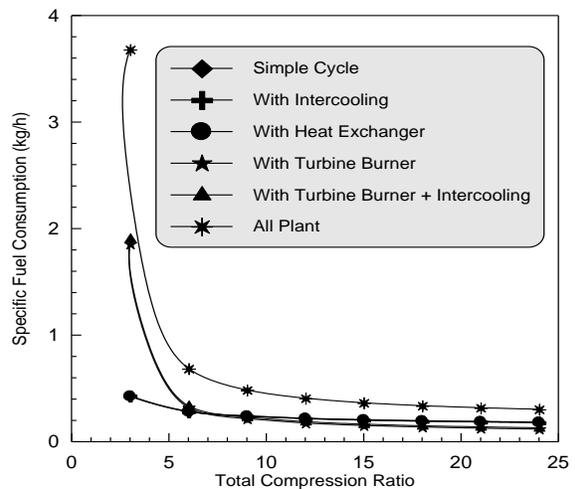


Fig. 8. Specific Fuel Consumption versus Pressure Ratio for all Enhancing Devices and Simple Cycle

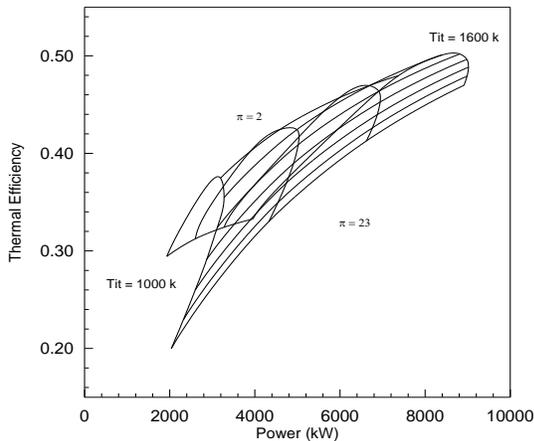


Fig. 9. Thermal Efficiency – Power Envelope for Different TIT and Pressure Ratios

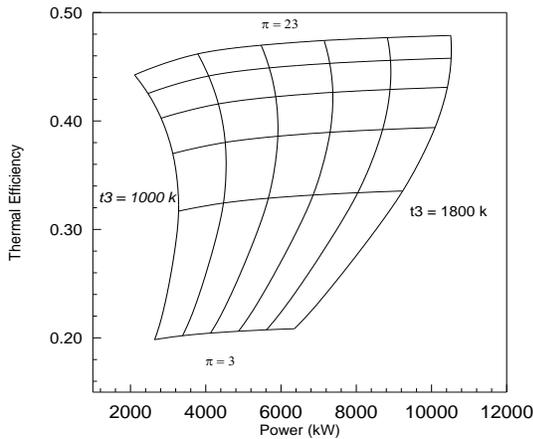


Fig. 10. Thermal Efficiency –Power Generalized Map

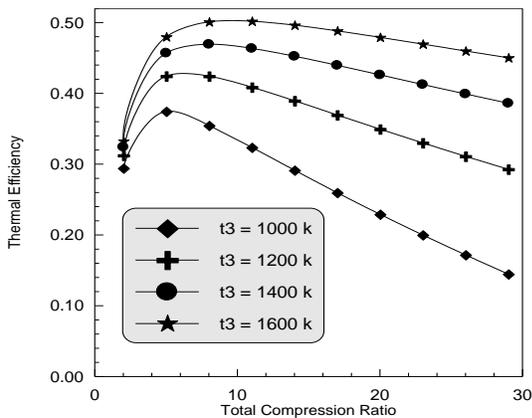


Fig. 11. Thermal Efficiency Dependence on Pressure Ratio for Different Turbine Inlet Temperature

Conclusions

A comparative study on the use of performance augmenting possibilities using the available enhancing devices was carried out. The following remarks were concluded:

1. The use of regeneration improves the efficiency of the plant but it is confined with a low range of compression ratio. The use of regeneration with other augmenting methods is necessary.
2. Intercooling between compressors leads to a low work shared to derive the compressor. The disadvantage of using Intercooling is from a thermodynamics point of view of losing heat because cooling. The influence of intercooling appears with high compression which is limited by the technology of compressor manufacturing.
3. The power is considerably augmented in case of using the turbine interstage burner. This is preferred in aircraft engine in order to increase the thrust loading ratio for a short period.
4. Implementing the Intercooling, turbine burner and regeneration improves the efficiency of the gas turbine power plant and depress the fuel consumption. A compromising investigation with cost of installation of these devices is then required.
5. The approach presented in this work leads to explore the decisive performance parameters in order to attain the maximum power with reasonable efficiency.

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Appendix

$$w_c = \frac{Cp_a(T_2 - T_1)}{\eta_c} \dots\dots\dots(1)$$

$$w_c = \frac{T_1 Cp_a \left(\frac{T_2}{T_1} - 1 \right)}{\eta_c} \dots\dots\dots(2)$$

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} = (\pi)^{\frac{\gamma-1}{\gamma}} \dots\dots\dots(3)$$

$$w_c = \frac{T_1 Cp_a \left((\pi)^{\frac{\gamma-1}{\gamma}} - 1 \right)}{\eta_c} \dots\dots\dots(4)$$

$$w_t = \eta_t T_{it} Cp_g \left(1 - \frac{1}{(\pi_{ipt})^{\frac{\gamma_g-1}{\gamma_g}}} \right) \dots\dots\dots(5)$$

$$Wn = Cp_g \left(T_{it} - \frac{Cp_a T_1}{Cp_g \eta_m} \left(\frac{(\pi_c)^{\frac{\gamma_a-1}{\gamma_c}} - 1}{\eta_c} \right) \right) \left(\eta_t \left(1 - \frac{1}{(\pi_{ipt})^{\frac{\gamma_s-1}{\gamma_g}}} \right) \right) \dots\dots\dots (6)$$

$$\eta_{th} = \frac{Cp_g \left(T_{it} - \frac{Cp_a T_1}{Cp_g \eta_m} \left(\frac{(\pi_c)^{\frac{\gamma_a-1}{\gamma_c}} - 1}{\eta_c} \right) \right) \left(\eta_t \left(1 - \frac{1}{(\pi_{ipt})^{\frac{\gamma_s-1}{\gamma_g}}} \right) \right)}{\frac{Cp_g Cp_a T_1}{Cp_g \eta_m} \left(\frac{(\pi_c)^{\frac{\gamma_a-1}{\gamma_c}} - 1}{\eta_c} \right)} \dots\dots\dots (7)$$

$$Wn = Cp_g \eta_t \left(1 - \frac{1}{(\pi_{ipt})^{\frac{\gamma_s-1}{\gamma_g}}} \right) \left(T_{it} - \frac{Cp_a T_1 \left(\frac{(\pi_c)^{\frac{\gamma_a-1}{\gamma_c}} - 1}{\eta_c} \right) \left(2 + (1-x) \left(\frac{(\pi_c)^{\frac{\gamma_a-1}{\gamma_c}} - 1}{\eta_c} \right) \right)}{Cp_g \eta_m \eta_t} \right) \dots\dots\dots (8)$$

$$\eta_{th} = \frac{Cp_g \eta_t \left(1 - \frac{1}{(\pi_{ipt})^{\frac{\gamma_s-1}{\gamma_g}}} \right) \left(T_{it} - \frac{2Cp_a T_1 \left(\frac{(\pi_c)^{\frac{\gamma_a-1}{\gamma_c}} - 1}{\eta_c} \right) \left(2 + (1-x) \left(\frac{(\pi_c)^{\frac{\gamma_a-1}{\gamma_c}} - 1}{\eta_c} \right) \right)}{Cp_g \eta_m \eta_t} \right)}{Cp_{g_m} \left(T_{it} - T_1 \left(1 + (1-x) \left(\frac{(\pi_c)^{\frac{\gamma_a-1}{\gamma_c}} - 1}{\eta_c} \right) \left(1 + \frac{(\pi_c)^{\frac{\gamma_a-1}{\gamma_c}} - 1}{\eta_c} \right) \right) \right)} \dots\dots\dots (9)$$

$$Wn = Cp_g \eta_t \left(T_{it} - \frac{Cp_a T_1}{Cp_g \eta_m} \left(\frac{(\pi_c)^{\frac{\gamma_a-1}{\gamma_c}} - 1}{\eta_c} \right) \right) \left(1 - \frac{1}{(\pi_{ipt})^{\frac{\gamma_s-1}{\gamma_g}}} \right) \dots\dots\dots (10)$$

$$\eta_{th} = \frac{Cp_g \eta_t \left(T_{it} - \frac{Cp_a T_1}{Cp_g \eta_m} \left(\frac{(\pi_c)^{\frac{\gamma_a-1}{\gamma_c}} - 1}{\eta_c} \right) \right) \left(1 - \frac{1}{(\pi_{ipt})^{\frac{\gamma_s-1}{\gamma_g}}} \right)}{Cp_{g_m} \left\{ T_{it} - \left(T_1 \left(1 + \frac{(\pi_c)^{\frac{\gamma_a-1}{\gamma_c}} - 1}{\eta_c} \right) \right) + \varepsilon \left[T_{it} - \frac{Cp_a T_1}{Cp_g \eta_m} \left(\frac{(\pi_c)^{\frac{\gamma_a-1}{\gamma_c}} - 1}{\eta_c} \right) \left(1 - \eta_t \left(1 - \frac{1}{(\pi_{ipt})^{\frac{\gamma_s-1}{\gamma_g}}} \right) \right) - T_{it} \left(1 + \frac{(\pi_c)^{\frac{\gamma_a-1}{\gamma_c}} - 1}{\eta_c} \right) \right] \right\}} \dots\dots\dots (11)$$

$$\eta_{th} = \frac{Cp_g T_{it} \eta_t \left(1 - \frac{1}{(\pi_{ipt})^{\frac{\gamma_s-1}{\gamma_g}}} \right)}{Cp_{g_m} \left\{ T_{it} - T_1 \left(1 + (1-x) \left(\frac{(\pi_c)^{\frac{\gamma_s-1}{\gamma_g}} - 1}{\eta_c} \right) \right) \left(1 + \frac{(\pi_c)^{\frac{\gamma_s-1}{\gamma_g}} - 1}{\eta_c} \right) \right\} + \frac{Cp_{g_m} Cp_a T_1}{Cp_g \eta_m} \left\{ \left(\frac{(\pi_c)^{\frac{\gamma_s-1}{\gamma_g}} - 1}{\eta_c} \right) \left(2 + (1-x) \left(\frac{(\pi_c)^{\frac{\gamma_s-1}{\gamma_g}} - 1}{\eta_c} \right) \right) \right\}} \dots\dots\dots (12)$$

$$\eta_{th} = \frac{Cp_g \eta_t T_{it} \left(1 - \frac{1}{(\pi_{ipt})^{\frac{\gamma_s-1}{\gamma_g}}} \right)}{Cp_{g_m} \left[\left(T_{it} - T_1 \left(1 + \frac{(\pi_c)^{\frac{\gamma_a-1}{\gamma_c}} - 1}{\eta_c} \right) \right) + \frac{Cp_a T_1}{Cp_g \eta_m} \left(\frac{(\pi_c)^{\frac{\gamma_a-1}{\gamma_c}} - 1}{\eta_c} \right) \right]} \dots\dots\dots (13)$$

$$\eta_{th} = \frac{Cp T_{it} \eta \left(1 - \frac{1}{(\pi_{ip})^{\frac{\gamma_s-1}{\gamma_g}}} \right)}{Cp_s \left\{ T_{it} - T_1 \left[\left(1 + (1-x) \left(\frac{(\pi_c)^{\frac{\gamma_s-1}{\gamma_g}} - 1}{\eta_c} \right) \right) \left(1 + \frac{(\pi_c)^{\frac{\gamma_s-1}{\gamma_g}} - 1}{\eta_c} \right) \right] - \varepsilon \left[\left(T_{it} \left(1 - \eta_t \left(1 - \frac{1}{(\pi_{ip})^{\frac{\gamma_s-1}{\gamma_g}}} \right) \right) \right) - T_{it} \left(1 + (1-x) \left(\frac{(\pi_c)^{\frac{\gamma_s-1}{\gamma_g}} - 1}{\eta_c} \right) \right) \left(1 + \frac{(\pi_c)^{\frac{\gamma_s-1}{\gamma_g}} - 1}{\eta_c} \right) \right] \right\}} \dots\dots\dots (14)$$