

## *Fuzzy logic Control of Continuous Stirred Tank Reactor*

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### **Abstract**

The mathematical modeling of continuous stirred tank reactor system is developed based on the mass and energy balances for the reactor and heating system. The system is studied by introducing step change in concentration, inlet flow, flow of heating fluid, inlet temperature and heating fluid temperature and measuring the temperature change in the reactor. In this paper, a fuzzy logic controller has been designed and evaluated. Through simulation study by using MATLAB, it has been shown that the proposed fuzzy logic controller has given an excellent tracking and regulation performance compared to that of the PID control system.

**Keywords:** Mathematical modeling of continuous stirred tank reactor, MATLAB simulation, Fuzzy logic, PID controller.

### **السيطرة على المفاعل ذو الخلط المستمر باستخدام منطق الدليل الغامض**

#### **الخلاصة**

تم دراسة الموديل الرياضي لمفاعل ذو الخلط المستمر بالاعتماد على موازنة المادة والطاقة للمفاعل. تم عرض النظام لتغيير درجي في التركيز ودرجة حرارة ومعدل جريان المادة الداخلة ودرجة حارة ومعدل جريان سائل التسخين وتم قياس درجة الحرارة داخل المفاعل. وكذلك تناول البحث تصميم طريقة سيطرة حديثة وهي طريقة الدليل الغامض وتم مقارنتها مع طريقة السيطرة التقليدية لجهاز التحكم من نوع تناسبي-تكاملي-تفاضلي باستخدام برنامج "الماتلاب". وقد أظهرت النتائج إن طريقة الدليل الغامض هي أفضل وذات كفاءة عالية مقارنة مع المسيطر التقليدي لأنها أقل تذبذب وأسرع بالوصول إلى القيمة المرغوبة.

**الكلمات الدالة:** التمثيل الرياضي لمفاعل ذو الخلط المستمر، المحاكاة باستخدام برنامج الماتلاب، الدليل الغامض، مسيطر تناسبي-تكاملي-تفاضلي.

E: Activation energy, (kJ/kmol).

e(t): Temperature error signal.

$F_i$ : Volumetric flow rate of the inlet stream, ( $m^3/hr$ ).

$F_j$ : Volumetric flow rate of the jacket, ( $m^3/hr$ ).

$\Delta H_r$ : Heat of reaction, (kJ/kmol).

$k_1$ : Reaction rate constant ( $min^{-1}$ ).

$K_c$ : Constant of Eq.(1).

$K_T$ : Constant of Eq.(1).

$K_3$  to  $K_7$ : Gains of energy balance in Eq. (4&6).

### **Nomenclature**

$C_A$ : Concentration of A in the reactor, ( $kmole/m^3$ ).

$C_{Ai}$ : Concentration of A in the inlet stream, ( $kmole/m^3$ ).

$C_{p_i}$ : Specific heat capacity of the inlet stream, (kJ/kg.K).

$C_{p_j}$ : Specific heat capacity of the jacket water, (kJ/kg.K).

$C_p$ : Specific heat capacity of the reacting mixture.

digital PID controllers (M'Saad *et al.* [6] and Cameron, Seborg [7]) and gain-scheduled fuzzy logic controller (Pramit [8]).

Czeczot [9] applied a balance-based adaptive control methodology and its application to the control of the continuous stirred tank reactor. This control methodology combines simplicity and generality, the simulation results prove very good control performance of the balance-based adaptive controller in comparison with the classical PI controller. Salehi & Mohammad [10] applied an adaptive fuzzy temperature controller for a class of continuous stirred tank, a fuzzy logic system is used to estimate the concentration dependent terms and other unknown system parameters in the control law, using temperature measurements. Prakash & Senthil [11] applied a nonlinear observer based model predictive controller continuous stirred tank reactor. The nonlinear state estimation is achieved by using fuzzy kalman filter and augmented state fuzzy kalman filter. The fuzzy kalman filter is used to estimate the reactor concentration and reactor temperature whereas the augmented state fuzzy kalman filter is used to estimate the reactor concentration, From the extensive simulation studies on the reactor, they infer that fuzzy kalman filter based nonlinear model predictive controller was able to achieve satisfactory servo performance. Soukkou *et al.* [12] applied a fuzzy optimal control methodology to the design of the feedback loops of an exothermic continuous stirred tank reactor system. Simulations demonstrate that the proposed robust has successfully met the design specifications. Galluzzo & Cosenza [13] considered a mixed feedback-feedforward control configuration and type-2 fuzzy logic

R: Universal gas constant, (kJ/kmol. K).

S: Laplace transform.

t : Time, (min.).

t<sub>d</sub>: Time delay, (min.).

T: Temperature of the outlet , (K).

T<sub>j</sub>: Temperature of the jacket, (K).

T<sub>ji</sub>: Inlet temperature of the jacket, (K).

U: Overall heat transfer coefficient, (kw/K.m<sup>2</sup>).

V: Volume of the reactor, (m<sup>3</sup>).

V<sub>j</sub> : Volume of the jacket, (m<sup>3</sup>).

ρ: Density of the outlet stream, (kg/m<sup>3</sup>).

ρ<sub>j</sub> :Density of the jacket water, (kg/m<sup>3</sup>).

τ: Time constant, (min.).

τ<sub>1</sub>: Concentration time constant, (min.) .

τ<sub>2</sub>: Thermal time constant, (min.).

τ<sub>3</sub>: Jacket time constant, (min.).

τ<sub>D</sub>: Derivative time constant, (min.).

τ<sub>I</sub>: Integral time constant, (min.).

μ<sub>A</sub>: Membership function.

## Introduction

Continuous stirred tank reactor system (CSTR) is a typical chemical reactor system with complex nonlinear dynamic characteristics. There has been considerable interest in its state estimation and real time control based on mathematical modeling. However, the lack of understanding of the dynamics of the process, the highly sensitive and nonlinear behavior of the reactor, has made difficult to develop a suitable control strategy. An efficient control of the CSTR can be achieved only through an accurate model [1].

For fuzzy control, the issue for designing robust controller using fuzzy logic technique has always been the key point. In most cases, fuzzy control is designed based on the use of heuristic information from human experts without using any plant model as a reference [2]. The control of continuous stirred tank reactor requires the application of advanced techniques such as neural network control ( Nikravesht *et al.* [3], Thompson and Kramer [4] and Seborg [5]),

Take the Laplace transform of Eq. (3):

$$T(s) = \frac{K_3}{(\tau_2 s + 1)} T_i(s) + \frac{K_4}{(\tau_2 s + 1)} T_j(s) + \frac{K_5}{(\tau_2 s + 1)} C_A(s) \dots\dots\dots (4)$$

The energy balance for the jacket becomes.

$$\frac{dT_j(t)}{dt} = \frac{F_j}{V_j} (T_{ji}(t) - T_j(t)) \dots\dots\dots (5)$$

Take the Laplace transform of Eq. (5):

$$T_j(s) = \frac{K_6}{(\tau_3 s + 1)} T_{ji}(s) + \frac{K_7}{(\tau_3 s + 1)} T(s) \dots\dots\dots (6)$$

**Fuzzy Logic Controller**

Fuzzy logic control is a methodology bridging artificial intelligence and traditional control theory. This methodology is usually applied in the only cases when accuracy is not of high necessity or importance. On the other hand, “Fuzzy Logic can address complex control problems, such as robotic arm movement, chemical or manufacturing process control, automobile transmission control with more precision and accuracy, in many cases, than traditional control techniques .... Fuzzy Logic is a methodology for expressing operational laws of a system in linguistic terms instead of mathematical equations.” Wide spread of the fuzzy control and high effectiveness of its applications in a great extend is determined by formalization opportunities of necessary behavior of a controller as a “fuzzy” (flexible) representation. This representation usually is formulated in the form of logical (fuzzy) rules under linguistic variables of a type “If A then B”<sup>(15)</sup>.

**Fuzzy Set Basic operation**

The theory of sets and the concept of a set itself constitute a foundation of

controllers they're considered for the temperature control of a non-isothermal continuous stirred tank reactor. The simulation results show that the proposed control configuration with type-2 FLCs can be an effective solution to a very difficult control problem and an alternative to the use of adaptive controllers.

**Mathematical Modeling of the Continuous Stirred Tank Reactor**

The dynamic behavior of the continuous stirred tank reactor has been modeled by the following set of nonlinear equations based on the mass and energy balances for the reactor and heating system. Heat and mass balance has been developed and comprehensively validated against the pilot plant. For the hydrolysis of ethyl acetate (A) forming ethanol & acetic acid with NaOH as a catalyst, the following differential equations describe the dynamics of the continuous stirred tank reactor <sup>[14]</sup>.

The mass balance equation of continuous stirred tank reactor becomes.

$$\begin{aligned} & \frac{V}{F_i + V K_c} \frac{d C_A(t)}{dt} + C_A(t) \\ &= \frac{F_i}{F_i + V K_c} C_{A_i}(t) \\ &- \frac{V K_T}{F_i + V K_c} T(t) \dots\dots\dots (1) \end{aligned}$$

Take the Laplace transform of Eq. (1):

$$C_A(s) = \frac{K_1}{(\tau_1 s + 1)} C_{A_i}(s) - \frac{K_2}{(\tau_1 s + 1)} \dots\dots (2)$$

The energy balance equation of continuous stirred tank reactor becomes.

$$\begin{aligned} & \rho C_p V \frac{dT(t)}{dt} + (\rho C_p F_i + UA - \Delta H_r V K_T) T(t) = \\ & \rho C_p F_i T_i(t) + U A T_j(t) + \Delta H_r V K_c C_A(t) \dots (3) \end{aligned}$$

In fuzzy set theory, the characteristic function is usually called the membership function.

**Design of Fuzzy Logic Controller**

The decision-making activities of a process operator in a regulation control task are shown in Figure (2); for the purposes of this work this activity is expressed as a fuzzy relationship or algorithm, relating significant observed variables to the control actions. The form of the decision rules employed depends on the process under control and the heuristics employed. In the case of single input-single output regulation tasks which are the subject of this study, the process operator is assumed to respond to the system error (E) and its rate of change (CE), the result of a control decision being a change in the control valve setting (CU). The resulting control system has a measurement and control action basis similar to the versatile proportional + integral control system used extensively in the process industry<sup>(17)</sup>. The first step in fuzzy logic is to convert the measured signal x (which might be the error signal in a control system) into a set of fuzzy variables. This is called fuzzy classification or fuzzification. It is done by giving values (these will be our fuzzy variables) to each of a set of membership functions. The values for each membership function are labeled  $\mu(x)$ , and are determined by the original measured signal x and the shapes of the membership functions. A common fuzzy classifier splits the signal x into five fuzzy levels as follows: -  
 a) LP: x is Large Positive  
 b) SP: x is Small Positive  
 c) Z: x is Zero  
 d) SN: x is Small Negative  
 e) LN: x is Large Negative

We can simply follow these steps in designing a fuzzy logic control:

modern mathematics. As far as one considers mathematical and simulation models of application problems, one deals with mathematics and the set theory at the base of mathematics. The space which fuzzy sets are working in is called the universal set. Then a fuzzy subset (A) of universal set (U) is characterized by a membership function ( $\mu_A(u)$ ) which is assigned to each element ( $U \ni u$ ). This function determines if the element of the universal set does or does not belong to this subset A. Hence the function may have two values: TRUE or FALSE or in numbers, 1 or 0.<sup>(16)</sup>

$$\mu_A(u) = \begin{cases} 1 & \text{if and only if } u \in A \\ 0 & \text{if and only if } u \notin A \dots \dots \dots \end{cases} \quad (7)$$

The main operations used are defined as follows<sup>(16)</sup>:

1-The intersection of the fuzzy subsets (A) and (B) of the universal with characteristic function define by:

$$\mu_{A \cap B}(u) = \min(\mu_A(u), \mu_B(u))$$

This corresponds to the logical “AND” operation.

2-The union of the fuzzy subsets (A) and (B) of the universe set (U) is denoted by: ( $A \cup B$ )

with characteristic function define by:

$$\mu_{A \cup B}(u) = \max(\mu_A(u), \mu_B(u))$$

This corresponds to the logical “OR” operation.

3-The complement of a fuzzy subset (A) of the universe set (U) is denoted by:

$$\mu_{\bar{A}}(u) = 1 - \mu_A(u)$$

This corresponds to the logical “NOT” operation.

$$\beta = \min. (\mu_E(u), \mu_{CE}(u))$$

5-The fuzzy decision rules are developed linguistically to do a particular control task and are implemented as a set of fuzzy conditional statements of the form:

“IF E is PB AND CE is NB THEN PS Action “

This form can be translated with the help of fuzzy sets definition into a new statement;

“IF PEB AND NCB THEN PUS”

The derivation of the fuzzy rules can be obtained directly from the phase-plane of error and its rate of change. Table (1) shows the fuzzy rules conclusions. The five fuzzy sets definition generates (25) rules fuzzy controller.

7- Choice of the defuzzification procedure. The defuzzification goal in Mamdani type fuzzy controllers is to produce a crisp output taking the fuzzy output obtained after rules processing. The center of gravity (COG) method is used.

8- Fuzzy Controller program: The fuzzy controller can be programmed in C, Fortran, Basic, MATLAB, or virtually any other programming language. Suppose that we let the computer variable (er) denote  $E(t)$ , which we call the first input, and (de) denote  $CE(t)$ , which we will call the second input. Using these definitions, consider the program for a fuzzy controller output given its two inputs:

- For  $i=1$  to 200
- Multiplying  $e(i)$  and  $der(i)$  by a suitable scale factor to get  $er(i)$  and  $de(i)$ . (inputs of fuzzy controller).
- Compute  $\mu_1$  and  $\mu_2$ . (Find the values of all membership

1- Choose a suitable scaled universe of set (U) of;

$$L \leq (E_i) \leq L$$

$$-L \leq (CE_i) \leq L$$

Where L and  $-L$  represent the positive and negative ends respectively of this universe which is quantified into equally spaced levels in between those two ends.  $E_i$  and  $CE_i$  represent the error and its rate of change for the same instant (i). The calculation of the error and its rate of change, from the fuzzy logic control point of view the calculations of error (E) and its rate of change (CE) are as below:

$$E_i = (\text{Measured value})_i - \text{Set value}$$

$$CE_i = \text{Instant error} - \text{Previous error}$$

The error definition here is the opposite of that used in other control theory and the cause is just to keep the linguistic that the fuzzy rules have already built on.

2-Both  $E_i$ ,  $CE_i$  and  $u$  are multiplied by a scale factor of the universe of set to ensure mapping their values into suitable intervals that belong to each one, also this scale factor helps to simplify handling the numerical values of all variable.

3-Choose a membership function, such as number of classes to describe all the values of the linguistic variable on the universe, the position of different membership functions on the universe of discourse, the width of the membership functions and the shape of a particular membership function.

4-Calculate the applicability degree. At this the degree to which the whole condition part (all the inputs) satisfies the rule is calculated. This degree is called the degree of applicability of the condition part. It is denoted as  $\beta$  :

blades. The stirrer operates with range of (0-250) rpm.

Three glass rectangular vessels of two liters were used as feeding tanks. These were fitted at the top of reactor and transfer of liquids by gravity to the reactor. All the transport tubes of the system to the reactor are 12 mm inside diameter and are made of stainless steel to prevent any chemical corrosion. Flow rates were measured by independently calibrated rotameters. The range of flow of each rotameter is (0 – 40 L/hr.) of water at 20<sup>0</sup> C.

The reactor was heated by hot water, which flows through the jacket around the reactor by using a small pump. The pump was capable of handling about (40 L/hr.) water and was heated by an external electrical heater. The electrical heater was provided with a digital screen to view the temperature.

Thermocouple (3B37-T-03) was installed to measure reactor temperature, the input range of this thermocouple (-100<sup>0</sup> C to +400<sup>0</sup> C) and the output ranges (0 – 10 mV), this signals were converted through transmitter and then fed to the computer through Analog – digital interface.

## Results and Discussion

### Open Loop System

In order to obtain a model for each process, the input and output data of the open loop system are needed for the evaluation purpose. The data is obtained from experiments done on the real process plant. The experimental results of the open loop responses in the reactor temperature for a step change in step change in ( $C_A, F_J, F, T, T_J$ ) are shown in the figures (3) to (6).

The mathematical model equations of continuous stirred tank reactor were solved using MATLAB software. The results values for model parameters are as follows:

functions given the values for (er) and (de).

- Compute  $\mu = \min(\mu_1, \mu_2)$ . (Find the values for the premise membership functions for a given (er) and (de).using the minimum operation).
- Compute "du"
- Let  $w$  (no. of rule)=0,  $du$ (no. of rule).=0
- After applying these six steps on our 25 rules, then calculate COG  
→  $num. = \sum [du * \mu] / \sum [\mu]$
- Output  $u$  crisp= $u(i) + COG * suitable$  scale factor for FLC output
- $T(i+1) = T(i+1) + u$

### Simulation of Control Methods

The simulation technique is based on the software tool MATLAB to solve the ordinary differential equations which represent the system behavior. During the digital simulation of the two methods of control, the controlled variable (T) is calculated and from the response of this variable we find the best condition of the control system.

The open loop responses to step change in ( $C_A, F_J, F, T, T_J$ ) are calculated from MATLAB program. The closed loop response for Fuzzy Logic controller is calculated by applying the steps in the simulation section, and software tool MATLAB to obtain the closed loop response to a step change in the mentioned variables.

### Experimental Work

The experiments were carried out in a laboratory scale continuous stirred tank reactor. The laboratory plant consists of 1 liter glass reactor with constant volume byproduct overflow out of the vessel. Surrounded by a glass jacket as shown in figure (1), it also equipped with the stirrer of stainless steel which has two-

controlling the continuous stirred tank reactor system using fuzzy logic by shorting the time requires for reaching the set point and eliminating the oscillation in the response.

### Conclusions

From the present study, the following conclusions are drawn regarding the control of continuous stirred tank reactor. The process identification procedure using process reaction curve methods showed that the system was described as first-order lag system with dead time. The results have shown priority of the fuzzy logic controller compared with the classical controller (PID). The comparison between the experimental data and simulation results gave a little deviation. It was considered that the differences were due to some reasons like the unisolated system and the non-linearity of the system

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$\tau_2=12.8\text{min}$ ,  $\tau_3=3.53\text{min}$ ,  $K_3=0.97$ ,  $K_4=0.028$ ,  $K_5=0.033$ ,  $K_6=0.08$  and  $K_7=1.76$

The transfer function of the reactor can represent as the following equation and Table (2) gives the value of these parameters:

$$G(s) = \frac{21.69e^{-0.36s}}{12.8s + 1} \dots\dots\dots(7)$$

Simulation was also used to calculate the controller parameters. These parameters were determined using Cohen-Coon method. The simulated reactor temperature responses are also shown in the same figures for comparison. The Mean Square Error between the simulated and experimental results are (0.077), (0.0743), (0.0595) & (0.0936) respectively. This deviation between the actual and simulated response arises from the linearization inherent in the transfer function model. However, the responses with simulated and experimental are not significantly different.

### Fuzzy Logic Controller

The fuzzy controller presented is applied for the continuous stirred tank reactor system. In order to certain the advantages offered by the fuzzy control strategy, simulation results are also presented for PID controller. A simulation study was carried out to establish the effectiveness of the proposed methods in controlling the reactor temperature and to predict the dynamic process behavior with tuning the parameters of the controllers.

The results obtained for the control system are shown in the figures (7) to (10). The results obtained with the fuzzy controller were much better than conventional method. The fuzzy controller gave good control at all operating points with a rapid response and small amount of overshoot. These responses show the improvement in

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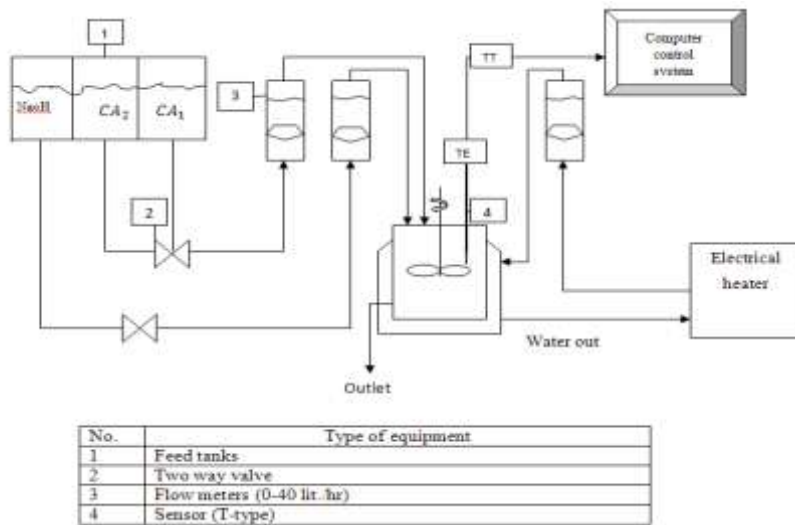


Figure (1): Schematic Diagram of the Experimental CSTR Process.

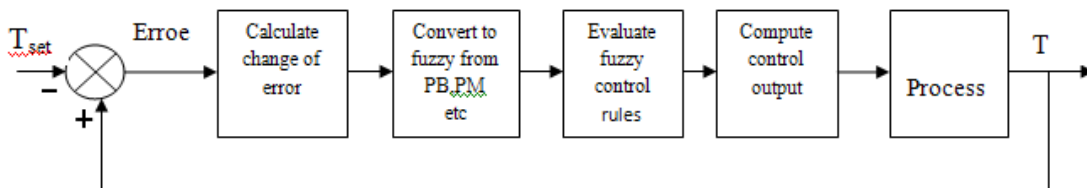


Figure (2) Block Diagram of the Fuzzy Control System

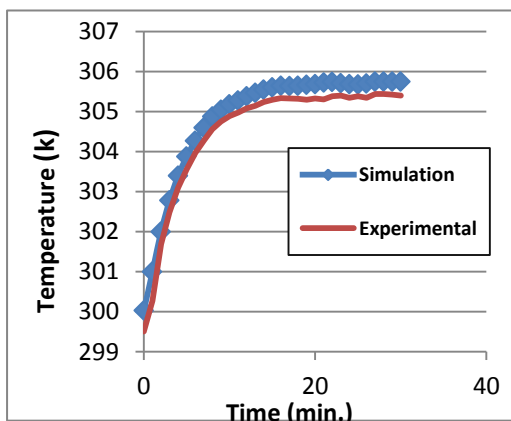


Figure (3): Comparison between the open-loop simulated and experimental temperature responses to step change in jacket flow rate from 10 to 30 L/hr.

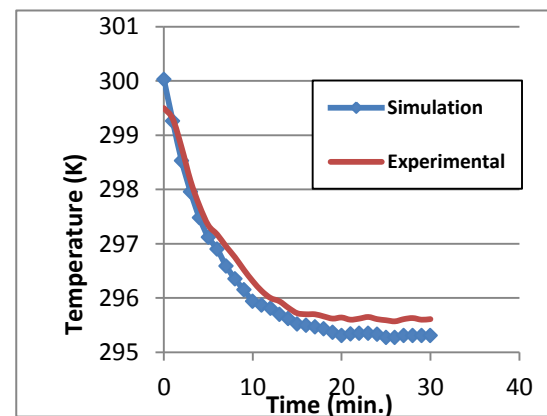


Figure (4): Comparison between the open-loop simulated and experimental temperature responses to step change in jacket temperature from 313 to 323 K.

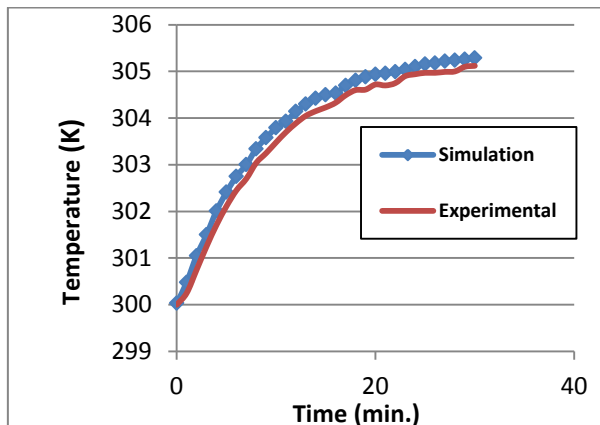


Figure (4): Comparison between the open-loop simulated and experimental temperature responses to step change in jacket temperature from 313 to 323 K.

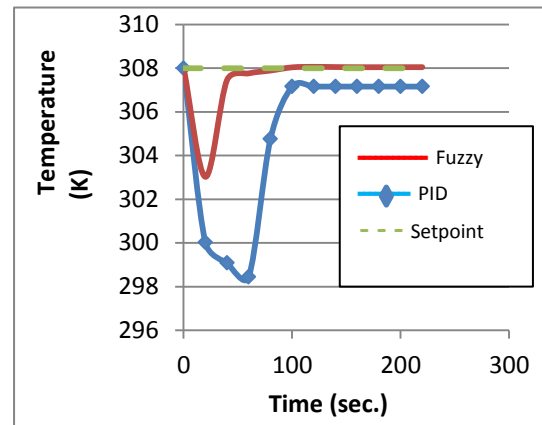


Figure (7) Comparison between conventional control and fuzzy logic control to a step change in jacket flow from 10 to 30 L/hr

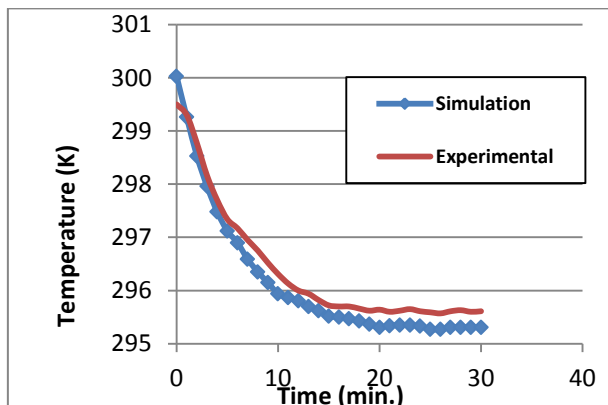


Figure (5): Comparison between the open-loop simulated and experimental temperature responses to step change in feed flow rate from 4.5 to 9 L/hr.

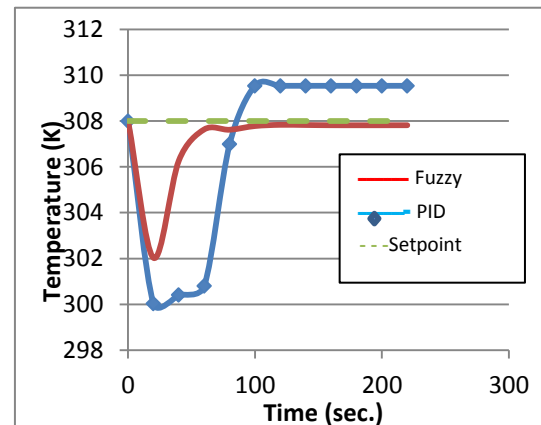


Figure (8) Comparison between conventional control and fuzzy logic control to a step change in feed temperature from 285 to 293K

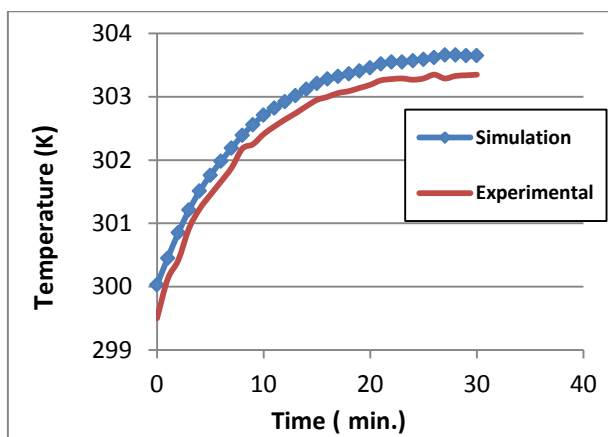


Figure (6): Comparison between the open-loop simulated and experimental temperature responses to step change in feed temperature from 285 to 293 K.

**Table (1): Fuzzy logic controller rules for CSTR**

CE \ E	PEB	PES	ZE	NES	NEB
PCB	NUB	NUB	NUB	NUS	ZU
PCS	NUB	NUS	NUS	ZU	PUS
ZC	NUB	NUS	ZU	PUS	PUB
NCS	NUS	ZU	PUS	PUS	PUB
NCB	ZU	PUS	PUB	PUB	PUB

**Table (2): Comparison between theoretical and experimental parameters of transfer function of CSTR.**

parameters	Theoretical	Experimental	Error
$K_P, K.m^3 / kmol$	21.69	21.54	0.15
$t_d, (min)$	0.36	0.5	0.15
$\tau, (min)$	12.8	18.92	6.12