Experimental Study of the Effect of Semi-Apical Angle on Initial Peak Load and Plastic Work for Nonmetallic Tubes

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Abstract
This article investigate the effect of semi-apical angle on the collapse load characteristics of thin truncated circular cones during axial crumpling. The PVC tubes axial crumpling resistance (the relationship between crumpling force and axial deflection) is described with mathematical models, which are derived depending on previous research and experimental present work. The mathematical models for present work gave good agreement with experimental results. The study showed that the values of initial peak load and plastic work decrease with increasing the semi-apical angle degree at constant thickness tubes and constant large diameter of end bottom of tubes and constant length.

Key words: Thin-tubes, axial buckling, plastic work of shells.

دراسة تجريبية لتأثير الزاوية النصف قممية على قيمة الحمل الابتدائي الأقصى والشغل اللدن
للأنابيب اللامعدنية

الخلاصة
دراسة تجريبية للأنابيب اللامعدنية تم فيها توضيح تأثير الزاوية النصف قممية على خصائص حمل الانهيار للأنابيب المخروطية المقطوعة ذات المقاطع الدائرية تحت تأثير حمل محوري. سلسلة من الاختبارات أجريت على النماذج بدرجة حرارة الغرفة تم من خلالها عرض نتائج حمل الأنبوب مع الاختزال الطولي النماذج في هذه الدراسة. تم مقارنة النتائج العملية للعمل التجريبي الحالي مع نتائج البحوث السابقة وأعطت توافق جيد مع نتائج الحالية. كذلك تم مقارنة النتائج العملية مع الموديل الرياضي المقترح وكانت النتائج النظرية للموديل المقترح قريبة جداً من النتائج العملية. تم التوصل في هذه الدراسة أن قيمة الحمل الابتدائي الأقصى والشغل اللدن يقل بزيادة الزاوية النصف قممية عند ثبوت السمك والقطر الكبير للنهاية الأنابيب المخروطية.

Notation
Dm mean diameter of circular tube
D mean outside diameter of frusta tube
Dt large diameter of end bottom of thin tube
ds small diameter of end top of thin tube
t thickness of thin tube
L length of thin tube
A cross-sectional area of circular tube
A’ cross-sectional area of frusta tube
Pmax experimental initial peak load
P theoretical initial peak load
σy yield stress
Wexp experimental plastic work
W theoretical plastic work
\[ \delta \] reduction in axial length or (deflection)
\[ \theta \] semi-apical angle of thin tube

**Introduction**

The crash behaviors of the thin-walled tubes during axial crushing and their axial resistances have aroused a lot of interesting. Previous researchers [1-7], have thoroughly illustrated the crash behaviors of different types of thin–walled tubes and developed a series of mathematical relations to describe or predict the axial crumpling load and the plastic deformation. The thin tubes become widely used in more applications of Engineering such as mechanical engineering and civil engineering because this structure (thin tubes) used to absorb energy and concrete reinforcement [8-10]. The research [8], recently explored the use of FRP jacket for confinement of the critical region of concrete-filled steel tubes and this study profit that jacket inward buckling deformation of steel tube is prevented by concrete core while the outward buckling deformation is prevented by the FRP the thin tube of steel.

The researches [11-13] applied different modes of deriving the mathematical equations that accurately predict the axial forces of different section tubes and profit that the sections of circular tubes is the best devices to absorb energy.

Axial crushing of frusta tubes has long been the subject of extensive research [14-17], these researches had studied the effect of the deformation on energy absorbing and mean axial loading for different materials such as steel and aluminum as well as the nonmetallic tube like the rigid PVC.

In the present work, experimental study on Polyvinylchloride frusta tubes compressed statically and axially to investigate the effect of semi-apical angle on the initial peak load and plastic work by deformation of material at constant thickness of tubes.

**Experimental Work and Results**

The axial compression of PVC circular and frusta tubes was carried out between parallel steel platens by ELE ADR-Auto 250/25 cemen compression test machine. The test material used was commercial rigid polyvinylchloride PVC [17] this material is widely used in engineering applications in mechanical and civil engineering. The stress-strain curve of which as obtained from static tensile test is given in Fig(1). From this curve, modeled as elastic-plastic behavior material, the initial value of yield stress was estimated to be 0.069 KN.mm^-2. The tests carried out at a crosshead speed of 10mm/min or over initial compression strain rate of (0.00133 sec^-1).

Details relating to the specimens dimensions are presented in table (1) refer to the geometrical representation of Figs(2a) and (2b). The change by degree of semi-apical angle (\( \theta \)) gives transform with geometrical shapes from circular tube at \( \theta = 0^\circ \) to truncated circular cones(Frusta) at \( 0 < \theta \leq 10^\circ \). The thickness (t) and axial length (L) of specimens is constant as shown in table (1). The values of large diameter (D_L) of end bottom of thin tubes is constant and The small diameter (d_s) of end top of thin tubes is change. All specimens tested at constant temperature.

The variation of collapsing load with amount of reduction in axial length to all specimens at failure were obtained using autographic recorder as shown in Figs(3-7) and the stages of deformation for specimens(reduction in axial length) observed in Figs(10a) and (10b). The values of the internal plastic work (\( W_{Exp} \)) and initial peak load (\( P_{max} \)) measuring from the load – deflection curve as shown in...
The value of the reduction of axial length at failure for all specimens test equal (83 mm).

The experimental plastic work equal the area under the curve and can be calculated by multiplying the average buckling load with reduction in axial length (δ).

### Theoretical Models

#### The Work Done by Plastic Deformation

Based on the experimental present work and previous researches for the PVC circular tubes, we proved that the value of plastic work by deformation of material is function many variables such as yield stress, thickness, mean diameter and the reduction in axial length, see equation (1). Also by the same method for the PVC frusta tubes, we proved that the value of plastic work depends on semi-apical angle, yield stress, thickness, ratio between small diameter of top end tube and mean diameter and the reduction in axial length, see equations (2).

In this analysis the assumption made for the collapse of thin circular and frusta tubes are the material is perfect plastic behavior and the energy dissipated by friction is neglected.

\[ W = R(\sigma_y, t, D_m, \delta) \]  

\[ W = q \tan \theta \{ \sigma_y, t, (d_s/D) \delta \} \]

Where constant material \( R = 1.85 \) for experimental present work.

\[ q = 2.04 \] Where the semi-apical angle(θ)=2.5°

\[ q = 4.49 \] Where the semi-apical angle(θ)=5°

\[ q = 9.63 \] Where the semi-apical angle(θ)=7.5°

\[ q = 13.94 \] Where the semi-apical angle(θ)=10°

#### Peak Load

For the PVC circular and frusta tubes the value of initial peak load can be calculated from equation (3) and (4) respectively.

\[ P = \sigma_y A \]  

\[ P = \sigma_y A' \]

Where the mean diameter \( D_m = (D - t) \).

\[ A = \pi D_m t \]

\[ A' = \pi D t \]

Where the mean outside diameter \( D = (D + d_s/2) \).

### Discussion and Conclusions

Mechanism of plastic collapse and mode deformation, and assuming the material is perfect plastic behavior, allowed the prediction of the value work done by plastic deformation and initial peak load for PVC circular and frusta tubes. Table(1) shows the values of work done and initial peak load for rigid PVC circular and frusta tubes.

The values of initial peak load computed from equations (3) and (4) these equations gave good a agreement with the experimental results for specimens of circular and frusta tubes respectively while the values of internal plastic work calculated from proposed mathematical models by equations (1) and (2) for circular and frusta tubes respectively and gave good agreement with experimental results and previous researches.

The fluctuation of buckling load for frusta tubes increase with progressing the reduction in axial length because the cross-sectional area of frusta will increase with the reduction in axial length and the collapse buckling depend on the mode of deformation. As shown in Figs(4-7).
Fig(3) illustrate that the values of the maximum initial peak load and plastic work carried out when the semi-apical angle(θ)=0 this value refer to that the geometrical shape of specimen is circular tube then the values of initial peak load and plastic work decrease with increasing the semi-apical angle with range(0<θ≤10) as shown in table(1), this range refer to that the geometrical shape is frusta tube(truncated circular cones).[16-17].

References
7. Yuh-Shiou Tai1, Mei-Yun, Huang, Hsuan-Teh, Hu." Numerical Study in Axial Impact of Thin-walled Circular Tubes Made of High Strength Steel"The international Society of offshore and polar Engineers(ISOPE),Oska Japan21-26,(2009).
15. YuchengLiu,Michael.,"Simplified Modeling of Thin-Walled Tubes


Fig (1): Stress-strain curve for rigid Polyvinylchloride (PVC).

Fig (2): (a) Diagram Section of circular tubes. (b) Diagram section of frusta tubes

Fig. (3) Load deflection curves for specimen (1)

Fig. (4) Load deflection curves for specimen (2)
Fig(5): Load-deflection curves for specimen (3).

Fig (6) Load deflection curves for specimen (4)

Fig. (7) Load deflection curves for specimen (5)

Fig. (8) Load deflection curves for specimen (6)

Fig(9): Variation of plastic work with semi-apical angle of specimens
Fig(10): (a) The deformation stages (reduction in axial length or deflection) for circular tube. (b) The deformation stages for frusta tube.

Table(1): The variation of values of plastic work and initial peak load with semi-apical angle.

<table>
<thead>
<tr>
<th>No</th>
<th>Semi-apical angle (θ) (degree)</th>
<th>shape</th>
<th>Axial Length (L) (mm)</th>
<th>Thickness (t) (mm)</th>
<th>diameter</th>
<th>Top narrow end diameter (D1) (mm)</th>
<th>Bottom large end diameter (D2) (mm)</th>
<th>Mean outside diameter (D3) (mm)</th>
<th>Experimental Work done (Wexp) (KN.mm)</th>
<th>Theoretical Work done (WT,Nm)</th>
<th>Experimental Initial peak load (Pmax) (KN)</th>
<th>Theoretical Initial peak load (pKN)</th>
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<td>59.1</td>
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<td>838.3</td>
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<tr>
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<td>59.1</td>
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