**Theoretical Study of the Compound Parabolic Trough Solar Collector**

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**Abstract**

Theoretical design of compound parabolic trough solar collector (CPC) without tracking is presented in this work. The thermal efficiency is obtained by using FORTRAN 90 program. The thermal efficiency is between (60-67)% at mass flow rate between (0.02-0.03) kg/s at concentration ratio of (3.8) without need to tracking system. The total and diffused radiation is calculated for Tikrit city by using theoretical equations. Good agreement between present work and the previous work.

**Keywords:** Solar energy, Solar water heater, CPC collector

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**Nomenclature**

- $A_a$: aperture area [m$^2$]  
- $A_{abs}$: absorber area [m$^2$]  
- $A_{con}$: concentrator area [m$^2$]  
- $C$: concentration ratio [dimensionless]  
- $C_f$: specific heat of fluid [J/kg. °C]  
- $d$: size of external aperture [m]  
- $D$: size of entrance aperture [m]  
- $D_{i, int}$: internal diameter of absorber tube [m]  
- $D_{e, ext}$: external diameter of absorber [m]  
- $F'$: efficiency factor of collector [dimensionless]  
- $F_R$: heat removal factor of collector [dimensionless]  
- $H$: height of concentrator [m]  
- $h_{c,i}$: convective heat transfer coefficient between receiver tube and fluid [W/m$^2$.°C]  
- $I_a$: absorbed radiation [W/m$^2$]  
- $I_{CPC}$: insolation [W/m$^2$]  
- $I_d$: diffused solar radiation [W/m$^2$]  
- $I_T$: total solar radiation [W/m$^2$]  
- $K$: conductivity of water [W/m.°C]  
- $L$: collector length [m]  
- $M$: mass flow rate [Kg/hr]  
- $n$: average number of reflections  
- $N$: number of absorber tube  
- $Nu$: Nusselt number of fluid inside absorber tube [dimensionless]  
- $Q_u$: useful energy [W]  
- $Re$: Reynolds number of fluid [dimensionless]  
- $T_a$: ambient temperature [°C]  
- $T_r$: receiver temperature [°C]
\[ \Delta T_f \text{ the temperature deference between the inlet and outlet fluid [°C]} \]
\[ T_{f,i} \text{ inlet fluid temperature [°C]} \]
\[ T_{f,o} \text{ outlet fluid temperature [°C]} \]
\[ T_{f,m} \text{ mean fluid temperature [°C]} \]
\[ U_L \text{ overall heat loss coefficient [W/m}^2\text{.°C]} \]

\textbf{Greek symbols}
\[ \alpha_r \text{ absorbance [dimensionless]} \]
\[ \varepsilon_r \text{ emissivity of absorber surface [dimensionless]} \]
\[ \theta_{max} \text{ acceptance angle [rad]} \]
\[ \eta \text{ thermal efficiency [dimensionless]} \]
\[ \rho \text{ reflectivity [dimensionless]} \]
\[ \tau_{CPC} \text{ effective transmissivity of CPC} \]
\[ \tau_{cover} \text{ transmissivity of glass cover} \]
\[ \gamma \text{ correction factor for diffuse radiation} \]

\textbf{Introduction}
Focusing collector is advice to collect solar energy with high intensity of solar radiation on the energy absorbing surface. Such collectors use optical system in the form of reflectors or refractors.

A focusing collector is a special form of a flat plate collector modified by introducing a reflecting (or reflecting) surface (concentrator) between the solar radiations and the absorber. Focusing collectors can have radiation increase from low value of 1,5 to 2, high values of the order of 10, 000.\cite{1}

One type of focusing collectors is a compound parabolic collector (CPC) or Winston collector as shown in figure (1). The CPC consists of two parabolic reflectors which funnel the incident solar radiation on to the absorber. The right and left halves belong to different parabolas. Each parabola is passing through the focus of the other parabola.

The distance between two focuses is the absorber. The CPC can be used in non-tracking mode with seasonal tilt adjustments and can provide concentration ratios in the range of (3-7).\cite{1}

Flat plate collectors have been widely used for applications that demand below 90°C and large amount of research efforts are already made. For medium temperature range (90–300°C) applications, concentrating type collectors are suitable, which are under investigation.\cite{1}

In this work a compound parabolic trough solar collector (CPC) is designed and studies theoretically. The dimensions of the designed model of CPC solar collector will be calculated by the using the equations which are explained later and the specifications of it will be selected.

Our work will cover (3-7) concentration ratio and the CPC type collector is suitable work with this area without need to the tracking system, but the other types of solar collectors needs the tracking system in additional to auxiliary systems and this will complicate the design and mean additional cost.

The thermal efficiency of the CPC collector in addition to the difference between inlet and outlet will study in this research.

A FORTRAN 90 program will build to deal with the calculation of the thermal efficiency of the CPC collector.

F. Bloisi et. al\cite{2} study four type of (CPC) collector. The four type of CPC collector are different in shape of absorber. The researchers study the effect of acceptance angle, height and width to the design of collector.

Zaki et. al.\cite{3} report that thermal losses from the CPC collector, due to a smaller absorber surface area, were
significantly reduced resulting in an increased thermal efficiency.

Norton et. al. [4] in a study investigating possible rural applications for the Compound Parabolic Concentrator (CPC) suggest the incorporation of a basin type still with an inverted absorber line-axis asymmetric CPC. The inverted absorber configuration can achieve higher temperatures by minimizing thermal losses by convection suppression.

Lixi Zhang et. al.[5] designed a new solar-heated generation system with capacity of 10kW. The CPC solar energy collector array is used as the main heat source (with concentration ratio equal 5), and the gas boiler as the assistant heat source. The shape of the CPC solar collector is designed, and the thermal efficiency is analyzed, and the collector array is ranged suitably. Finally, the economical benefit of the system is discussed.

Luisa I. and Feliciano-Cruz[6] reported the design of a simulation model for the analysis and performance evaluation of a Solar Thermal Power Plant in Puerto Rico and suggests the use of the Compound Parabolic Concentrator (CPC) as the solar collector of choice. The solar array would consist of 80 series collectors (1.52 m wide, 12 m long with a height of 1.97 m and a reflector area of 49.6 m$^2$).

Theory
A two dimensional CPC as shown in fig.(2) consist of two distinct parabolic segments placed in such a manner that the focus of one parabola placed on the other. The axes of two parabolic segments are oriented away from the CPC axis by the acceptance angle $\theta_{max}$. The slope of the parabolic reflector surface at the entrance aperture is parallel to the CPC optical axis. Thus the solar rays entering the concentrator at the maximum acceptance angle are reflected tangentially to the surface of the absorber.

For the simple geometry it can be shown that [1],

$$\tan \theta_{max} = \frac{D + d}{2H} \quad \text{...............(1)}$$

Where D is the size of entrance aperture, d of exit aperture and H the height of concentrator of CPC:

$$\frac{D}{d} = \frac{1}{\sin \theta_{max}} \quad \text{...............(2)}$$

Using above equations

$$H = \frac{D(1 + \sin \theta_{max})}{2\tan \theta_{max}} \quad \text{...............(3)}$$

Rabl[7] has shown that the area of the concentrator or reflector, $A_{con}$, is related with the area of the apertures $A_a$, as

$$A_{con} = A_a (1 + \sin \theta_{max})$$

$$\frac{\cos \theta_{max}}{\sin^2 \theta_{max}} + \ln\left\{ \frac{(1 + \sin \theta_{max}) (1 + \cos \theta_{max})}{\sin \theta_{max} \cos \theta_{max} + (2 + 2\sin \theta_{max})\sqrt{2}} \right\} - \frac{\sqrt{2} \cos \theta_{max}}{(1 + \sin \theta_{max})\sqrt{2}}$$

$$\text{...............(4)}$$

Rabl[7] has also shown that the average number of reflection ,n, passing through a CPC inside is acceptance angle is given as

$$n = \frac{1}{2\sin \theta_{max}} \left( \frac{A_{con}}{A_a} \right) \left( \frac{1 + 2\sin \theta_{max}}{1 - \sin \theta_{max}} \right)$$

$$\text{...............(5)}$$
The effective transmissivity of CPC, \( \tau_{CPC} \), accounting for reflection loss inside the CPC depends on the specular reflectivity, \( \rho \), of CPC wall and the average number of reflections, \( n \), and is given as\(^{[1]} \):

\[
\tau_{CPC} = \rho^n \quad \text{..........................(6)}
\]

The useful energy \( Q_u \) can be calculated as was done earlier if we know the absorbed energy \( I_a \) and \( U_L \).

The insolation, \( I_{CPC} \) within the acceptance angle of CPC with concentration ratio, \( C \), is given as\(^{[1]} \):

\[
I_{CPC} = I_T \left( 1 - \frac{1}{C} \right) I_d \quad \text{...............(7)}
\]

Where \( I_T \) and \( I_d \) are the total and diffuse radiation respectively on the aperture plane. Now the absorbed radiation \( I_a \) in terms of \( I_{CPC} \) is\(^{[1]} \):

\[
I_a = I_{CPC} \tau_{cover} \tau_{CPC} \alpha_r \\
= I_T \tau_{cover} \tau_{CPC} \alpha_r \gamma \quad \text{.............(8)}
\]

\[
= 1 - \left( 1 - \frac{1}{C} \right) \frac{I_d}{I_T} \quad \text{...............(9)}
\]

Where \( \tau_{cover} \) = transmissivity of cover \\
\( \tau_{CPC} \) = effective transmissivity of CPC \\
\( \alpha_r \) = absorbtivity of receiver \\
\( \gamma \) = correction factor for diffuse radiation.

The empirical expression of \( U_L \) for a CPC with tubular absorber coated with selective coating, covered with concentric glass cover, space evacuated and the entire collector covered with a transparent cover is given as\(^{[6]} \):

\[
U_L = \left( 0.18 + 16.95 \varepsilon_r \right) \\
\left[ 0.212 + 0.00255T_a + \left( 0.00186 + 0.00012T_a \right) \left( T_r - T_a \right) \right] \quad \text{..........(10)}
\]

Where

\( T_r \) = ambient temperature, °C
\( T_a \) = absorber temperature, °C
\( \varepsilon_r \) = emissivity of absorber surface
\( U_L \) = collector heat loss coefficient, W/m²K of absorber area.

**Performance Analysis of a Compound Parabolic Concentrating Collector (CPC)**

A compound parabolic concentrating (CPC) collector is generally covered with a transparent cover and is tilted towards the south with long axis in the East-West direction. CPC is tilted in such a fashion that it receives both beam radiation within the acceptance angle. Since in a CPC, the acceptance angle (\( \theta_{max} \)) is large it receives both beam and diffuse radiation. The absorber or receiver can be of any shape but generally tubes are used which are selectively coated and attached to the bottom as shown in Fig. 3.

The expression for the rate of useful energy collection is given as\(^{[1]} \):

\[
Q_a = A_a F_R \left[ U_L \left( T_f - T_a \right) \right] \\
\]

Where:

\( A_a = DL \quad \text{..........................(12)} \)

\[
I_a = I_T \tau_{cover} \tau_{CPC} \alpha_r \gamma \quad \text{..................(13)}
\]
The heat removal efficiency factor is given as [1]:

\[ F_R = \frac{MC_f}{A_{abs}U_L} \left[ 1 - \exp\left( -\frac{A_{abs}U_L F'}{MC_f} \right) \right] \]

\[ \ldots \ldots (14) \]

Where:

\[ A_{abs} = dL \]

\[ \ldots \ldots (15) \]

The collector efficiency factor is given as [1]:

\[ F' = \frac{1}{U_L} \left( 1 - \frac{dU_L}{U_L} N\pi D_{abs,i} h_{ei} \right) \]

\[ \ldots \ldots (16) \]

Where

\[ h_{ei} = \text{heat transfer coefficient inside the tube} \]

\[ N_u = 0.023 \Re^{0.8} \Pr^{0.4} \]

\[ = \frac{h_{ei} D_{abs,i}}{K} \]

\[ \ldots \ldots (17) \]

The outlet fluid temperature is calculated from equation as:

\[ T_{f,o} = T_{f,i} + \frac{Q_u}{MC_f} \]

\[ \ldots \ldots (18) \]

\[ T_{f,m} = \frac{T_{f,i} + T_{f,o}}{2} \]

\[ \ldots \ldots (19) \]

\[ T_r = T_{f,m} + \frac{MC_f (T_{f,o} - T_{f,i})}{h_{ei} \pi D_{r,ext} L} \]

\[ \ldots \ldots (20) \]

Finally the efficiency of the collector can be calculated from equation as:

\[ \eta = \frac{Q_u}{A_u I_b} \]

\[ \ldots \ldots (22) \]

**Design the CPC Solar Collector**

In this design, the concentration ratio (C) of the CPC selected as 3.8, the size of external aperture (d) selected as 0.5 m, thus from equation (2) the half acceptance angle (θ_max) found to be 15.2°. Again from equation (2), the size of entrance aperture (D) can be obtained and to be 1.9 m.

The height of the CPC (H) can be obtained by the substitution of the above parameters (D, θ_max) in equation (3) can be obtained and will be 4.4m. The length of CPC is selected as 4 m.

The reflectivity (ρ) of the reflector is selected to be (97.4%) (SolaReflex thick foil material). The transmissivity of the glass cover is selected to be 0.95. Absorbity of the absorber is selected to be 0.95 and the emissivity is 0.9. The internal and external diameters of the absorber tube are selects to be (0.01m) (0.012) respectively and the number of absorber tubes is 2. Assume the CPC collector was directed to the south and its slop (β) is equal to 23° and 45° in summer and winter, respectively.

**Results and discussion**

The theoretical study was performed using FORTRAN 90 program depend on simple iteration technique has been used to determine the absorber temperature. The dimensions and the specifications of the collector were entered to the program in order to determine the theoretical thermal efficiency. The flow chart of the
The solar radiation is determined theoretically. The ambient temperature and inlet flow temperature is measured experimentally in the winter and summer seasons in Tikrit city.

Figs. (5,7) illustrated the variation of the thermal efficiency and the flow temperature differences and the ambient temperature within the daylight hours in the summer and winter seasons. These figures explain the rise of the mass flow rate and this give high efficiency ranging from (60) % to (67) % for the flow rate values in the range (0.02-0.03) kg/s. Also these figures show that the minimum difference between the inlet and the outlet temperatures at eight o'clock in the morning and then begin rising until mid-day then at twelve o'clock noon it begins descending until four o'clock afternoon the end of the test period. The temperature difference (ΔT) is in the range (60°C-43°C) for a mass flow rate range (0.02-0.03) kg/s.

Figs (6,8) illustrated the variation of the useful energy and the solar radiation during the daylight hours in the winter and summer seasons. The mass flow rate range is (0.02-0.03) kg/s. The figures show that the increase in the solar radiation leads to an increase in the useful energy and the increase in the mass flow rate leads to a decrease in the useful energy. Also the figures show that the useful energy range is (170-7000) W/m² and the total solar radiation range is (300-1100) W/m² and the diffused solar radiation range is (50-108) W/m².

Fig. (9) shows a comparison between the thermal efficiency of CPC of the present work with the theoretical and experimental thermal efficiency of the CPC of reference [8]. The dimensions and specifications of the CPC of reference [8] are entered to FORTRAN 90 program of the present work in order to obtain results that can be compared with the present work results. Also this figure show that for mass flow rate (0.0055) kg/s , the range of the thermal efficiency of present work is in the range (47-56) % and the range of the thermal efficiency of reference[8] is (49-59). This figure shows a good agreement between the present work and reference [8].

Conclusions
- The thermal efficiency range is (60) % to (67) % at mass flow rate range (0.02-0.03) kg/s.
- The increase in the solar radiation leads to increase in the useful energy and the increased of the mass flow rate leads to a decrease in the useful energy.

References
5. Lixi Zhang, Chunlei Li, "The Research of Solar-heated Generation System Using CPC Collector",...


Fig. (1) Compound Parabolic Concentrator

Fig. (2): Geometry of a compound parabolic concentrator.

Fig. (3): Schematic of CPC

Fig. (4) Flow chart to calculate theoretical thermal efficiency of CPC solar collector

Input Collector and System Parameter

Start

Calculation of solar radiation from equations in appendix A

SET \( T_r = T_{fi}, T_{fm} = T_{fi} \)

Calculate overall heat transfer coefficient \( U_l \)

Calculate water properties

Calculate \( Re, Nu, h \) for water

Calculate \( F', F_R \)

Calculate concentration ratio

Calculate acceptance angle

Calculate useful energy \( (Q_u) \)

Estimate \( T_m, T_{fmm} \)

No

\( T_m - T_r \leq 0.001 \)

Yes

Calculate The Collector Efficiency

End
Fig. (5) the variation of thermal efficiency and temperature difference and ambient temperature with hours of daylight in winter.

Fig. (6) the variation of useful energy and solar radiatior with hours of daylight in winter.

Fig. (7) the variation of thermal efficiency and temperature difference and ambient temperature with hours of daylight in summer.

Fig. (8) the variation of useful energy and solar radiation with hours of daylight in summer.

Fig. (9) The comparison between efficiency of present work with efficiency of ref.[8]