

Image Compression using Haar and Modified Haar Wavelet Transform

**Mohannad Abid Shehab
Ahmed
Assistant Lecturer**

**Haithem Abd Al-Raheem
Taha
Assistant Lecturer**

**Musab Tahseen Salah
Aldeen
Assistant Lecturer**

Electrical Eng. Dept.- Al-Mustansirya University

Abstract

Efficient image compression approaches can provide the best solutions to the recent growth of the data intensive and multimedia based applications. As presented in many papers the Haar matrix-based methods and wavelet analysis can be used in various areas of image processing such as edge detection, preserving, smoothing or filtering. In this paper, color image compression analysis and synthesis based on Haar and modified Haar is presented. The standard Haar wavelet transformation with $N=2$ is composed of a sequence of low-pass and high-pass filters, known as a filter bank, the vertical and horizontal Haar filters are composed to construct four 2-dimensional filters, such filters applied directly to the image to speed up the implementation of the Haar wavelet transform. Modified Haar technique is studied and implemented for odd based numbers i.e. ($N=3$ & $N=5$) to generate many solution sets, these sets are tested using the energy function or numerical method to get the optimum one.

The Haar transform is simple, efficient in memory usage due to high zero value spread (it can use sparse principle), and exactly reversible without the edge effects as compared to DCT (Discrete Cosine Transform). The implemented Matlab simulation results prove the effectiveness of DWT (Discrete Wave Transform) algorithms based on Haar and Modified Haar techniques in attaining an efficient compression ratio (C.R), achieving higher peak signal to noise ratio (PSNR), and the resulting images are of much smoother as compared to standard JPEG especially for high C.R.

A comparison between standard JPEG, Haar, and Modified Haar techniques is done finally which approves the highest capability of Modified Haar between others.

Keywords: Discrete Wavelet Transform, Haar, Modified Haar, Linear Matrix Algebra, Sparse matrix.

الخلاصة

هناك طرق كفوءة لضغط الصور حيث يمكن ان تجهزنا بحلول افضل لتزايد انتشار البيانات وكثرة تطبيقات الوسط المتعددة، كما ممثل في الكثير من البحوث فان طريقة هار المبنية على تحويل الموجة يمكن استعمالها في تطبيقات متنوعة للمعالجة الصوريه مثل كشف الحافات وتنعيمها. في هذا البحث تم تحليل و بناء منظومة كبس الصور الملونة باستعمال طريقة هار و طريقة هار المعدلة.

طريقة هار القياسية لتحويل الموجة مبنية على اساس $N=2$ وتوليد مرشحات الامرار الواطي ومرشحات الامرار العالي حيث تطبق هذه المرشحات مباشرة على الصورة لتسريع العمل وتنفيذه. طريقة هار المعدلة لتحويل

الموجة مبنية على اساس $N=3$ & $N=5$ لتوليد عدة مجاميع حلول ثم يتم استعمال مبداء الطاقة او استعمال الدوال العددية بالماتلاب لمعرفة الحل الافضل.

طرق هار بسيطة عدديا، كفاءة في استعمال الذاكرة بسبب انتشار الاصفار (يمكن استعمال مبداء التناثر sparse)، وهي طريقة معكوسة بنفس عناصر المصفوفة الاصلية. نتائج برنامج الماتلاب المقترح اثبتت فعالية تحويل الموجة المبني على طريقة هار القياسية و طريقة هار المعدلة في الحصول على نسبة ضغط اعلى وقدرة اشارة عالية نسبة الى الضوضاء مع اعطاء تعميم للصورة. مقارنة بين ال JPEG القياسي و طريقة هار القياسية و طريقة هار المعدلة تمت في نهاية البحث على بعض الصور المختلفة واثبتت ان طريقة هار المعدلة هي الافضل. الكلمات الدالة: تحويل الموجة، طريقة هار القياسية، طريقة هار المعدلة، المصفوفات المتناثرة.

Introduction

One of the important factors for image storage or transmission over any communication media is the image compression. Compression makes it possible for creating file sizes of manageable, storable and transmittable dimensions. A 4 MB image will take more than a minute to download using a 64kbps channel, whereas, if the image is compressed with a ratio of 10:1, it will have a size of 400KB and will take about 6 seconds to download.

Image compression techniques fall under two categories, namely, Lossless and Lossy. In Lossless techniques, the image can be reconstructed after compression, without any loss of data in the entire process. Lossy techniques, on the other hand, are irreversible, because, they involve performing quantization, which result in loss of data. Some of the commonly used techniques are transform coding, namely, Discrete Cosine Transform, Wavelet Transform, Gabor Transform etc, Vector Quantization, Segmentation and approximation methods, Spline approximation methods (Bilinear Interpolation/Regularisation), Fractal coding etc^[1].

The compression ratio (C.R) in lossy techniques is larger than that of lossless.

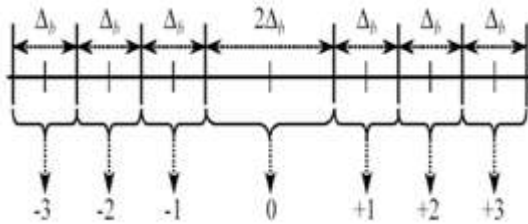
Quantization

It is a process used to reduce the number of bits needed to store the transformed coefficients by reducing the precision of those values. Since it is a many-to-one mapping, it is a lossy process and is the main source of compression in an encoder. Quantization can be performed on each individual coefficient, which is called Scalar Quantization (SQ). Quantization can also be applied on a group of coefficients together known as Vector Quantization (VQ). Both uniform and non-uniform quantizers can be used depending on the problems.

In this paper, a uniform scalar quantization with deadzone is used to quantize all the wavelet coefficients, for each sub-band b , a basic quantizer step size Δ_b is selected by the user and is used to quantize all the coefficients in that sub-band with the rule:

$$q = \text{sign}(y) \left\lfloor \frac{|y|}{\Delta_b} \right\rfloor \dots\dots\dots(1)$$

Where, y is the input to the quantizer, Δ_b is the quantizer step size, q is the resulting quantizer index. $\text{sign}(y)$ denotes the sign of y , $|y|$ denotes the absolute value of y ^[2].



The dequantization rule:

$$z = [q + r \cdot \text{sign}(q)] \Delta_b \quad \text{for } q \neq 0$$

$$z = 0 \quad \text{other wise}$$

Where r is the reconstruction bias,

$r = 0.5$ results in midpoint reconstruction (no bias)

$r < 0.5$ biases the reconstruction towards zero.

A typical value for $r = 0.375$.

Haar Wavelet

More recently, the wavelet transform has emerged as a cutting edge technology, within the field of image analysis. Wavelets are a mathematical tool for hierarchically decomposing functions. Though rooted in approximation theory, signal processing, and physics, wavelets have also recently been applied to many problems in Computer Graphics including image editing and compression, automatic level-of-detail control for editing and rendering curves and surfaces, surface reconstruction from contours and fast methods for solving simulation problems in 3D modeling, global illumination, and animation. Wavelet-based coding provides substantial improvements in picture quality at higher compression ratios.

Over the past few years, a variety of powerful and sophisticated wavelet-based schemes for image compression have been developed and implemented, the many advantages of wavelet based image compression are as listed below:

- Wavelet coding schemes at higher compression avoid blocking artifacts.

- They are better matched to the HVS (Human Visual System) characteristics.
- Compression with wavelets is scalable as the transform process can be applied to an image as many times as wanted and hence very high compression ratios can be achieved.
- Wavelet-based compression allows parametric gain control for image softening and sharpening.
- Wavelet-based coding is more robust under transmission and decoding errors, and also facilitates progressive transmission of images.
- Wavelet compression is very efficient at low bit rates.
- Wavelets provide an efficient decomposition of signals prior to compression [3].

In against, the disadvantages of wavelet are:

- The cost of computing DWT as compared to DCT may be higher.
- The large DWT basis functions or wavelet filters produce blurring and ringing noise near edge regions in images or video frames.
- Longer compression time.
- Lower than or comparable quality with JPEG at low compression rates.

The Haar wavelet is the simplest possible wavelet. The technical disadvantage of the Haar wavelet is that it is not continuous, and therefore not differentiable. This property can, however, be an advantage for the analysis of images and signals with sudden transitions, such as monitoring of tool, failure in machines.

The Haar wavelet's mother wavelet function $\psi(t)$ can be described as:

$$\Psi(t) = \begin{cases} 1 & 0 \leq t < 1/2 \\ -1 & 1/2 \leq t < 1 \\ 0 & \text{otherwise} \end{cases} \dots\dots\dots(2)$$

And its scaling function $\phi(t)$ can be described as:

$$\phi(t) = \begin{cases} 1 & 0 \leq t < 1 \\ 0 & \text{otherwise} \end{cases} \dots\dots\dots(3)$$

The Haar wavelet has several notable properties, the most useful property that can be extended to modify types, is the wavelet/scaling functions with different scale have a functional relationship ^[2]:

$$\phi(t) = \phi(2t) + \phi(2t - 1) \dots\dots\dots(4)$$

$$\psi(t) = \phi(2t) - \phi(2t - 1) \dots\dots\dots(5)$$

Standard Haar Wavelet Transform

The Haar wavelet transformation is composed of a sequence of low-pass and high-pass filters, known as a filter bank. The low pass filter performs an averaging/blurring operations, which is expressed as ^{[4][5]}.

$$L = \frac{1}{\sqrt{2}}(1\dots 1) \dots\dots\dots(6)$$

The high-pass filter performs a differencing operation and can be expressed as:

$$H = \frac{1}{2}(1 - 1) \dots\dots\dots(7)$$

The low and high filter's equations above, can be formulated simultaneously through four filters i.e., (LL, HL, LH, and HH) each of (2x2) adjacent pixels which are picked as group and assed .

In this transform, the bases of these 4-filters could be derived as follows:

- The horizontal low pass followed by the vertical low pass filter is equivalent to:

$$LL = \frac{1}{2} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \dots\dots\dots(8)$$

- The horizontal high pass filter followed by vertical low pass filter is:

$$LL = \frac{1}{2} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \begin{pmatrix} 1 & 1 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} +1 & +1 \\ -1 & -1 \end{pmatrix} \dots\dots(9)$$

- While the horizontal low pass filter followed by vertical high pass filter is equivalent:

$$LH = \frac{1}{2} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \begin{pmatrix} 1 & -1 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix} \dots\dots(10)$$

- Finally, the horizontal high pass filter followed by vertical high pass filter is:

$$HH = \frac{1}{2} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \begin{pmatrix} 1 & -1 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & -1 \\ -1 & +1 \end{pmatrix} \dots\dots(11)$$

The Haar Wavelet Transform is a vector transform, for [6] many steps can be modified and summarized by calculating:

$$r_{\text{new}} = W_1 W_2 W_3 r_{\text{old}} = W . r_{\text{old}} \dots\dots(12)$$

Where,

$$W_1 = \begin{bmatrix} 1/2 & 0 & 0 & 0 & 1/2 & 0 & 0 & 0 \\ 1/2 & 0 & 0 & 0 & -1/2 & 0 & 0 & 0 \\ 0 & 1/2 & 0 & 0 & 0 & 1/2 & 0 & 0 \\ 0 & 1/2 & 0 & 0 & 0 & -1/2 & 0 & 0 \\ 0 & 0 & 1/2 & 0 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 1/2 & 0 & 0 & 0 & -1/2 & 0 \\ 0 & 0 & 0 & 1/2 & 0 & 0 & 0 & 1/2 \\ 0 & 0 & 0 & 1/2 & 0 & 0 & 0 & -1/2 \end{bmatrix}$$

$$W_2 = \begin{bmatrix} 1/2 & 0 & 1/2 & 0 & 0 & 0 & 0 & 0 \\ 1/2 & 0 & -1/2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/2 & 0 & 1/2 & 0 & 0 & 0 & 0 \\ 0 & 1/2 & 0 & -1/2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$W_3 = \begin{bmatrix} 1/2 & 1/2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1/2 & -1/2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The above three matrices

W_1, W_2, W_3 represents the following operations:

- Dividing the entries into pairs
- Averaging these pairs
- Subtracts the average from the first entry

Let,

$$W = W_1 W_2 W_3 = \begin{bmatrix} 1/8 & 1/8 & 1/4 & 0 & 1/2 & 0 & 0 & 0 \\ 1/8 & 1/8 & 1/4 & 0 & -1/2 & 0 & 0 & 0 \\ 1/8 & 1/8 & -1/4 & 0 & 0 & 1/2 & 0 & 0 \\ 1/8 & 1/8 & -1/4 & 0 & 0 & -1/2 & 0 & 0 \\ 1/8 & -1/8 & 0 & 1/4 & 0 & 0 & 1/2 & 0 \\ 1/8 & -1/8 & 0 & 1/4 & 0 & 0 & -1/2 & 0 \\ 1/8 & -1/8 & 0 & -1/4 & 0 & 0 & 0 & 1/2 \\ 1/8 & -1/8 & 0 & -1/4 & 0 & 0 & 0 & -1/2 \end{bmatrix}$$

As a product of invertible matrices, W is also invertible. The inverse of W is given by:

$$W^{-1} = W_3^{-1} W_2^{-1} W_1^{-1} \dots\dots\dots(12)$$

The fact that the W is invertible allows to retrieve image from the compressed form using the relation:

$$r_{old} = W^{-1} r_{new} \dots\dots\dots(13)$$

The above summarized steps permits the using of matrix algebra manipulation and linear algebra to make the compression process faster, more efficient and easiest.

Suppose that A is a matrix corresponding to a certain image so the Haar transform can be carried out by performing the above operations on each row of the matrix A and then by repeating the same operations on the columns of the resulting matrix. The row-transformed matrix is AW and transforming the columns of AW is obtained by multiplying AW by the matrix W^T (the transpose of W). Thus, the Haar transform takes the matrix A and stores it as $W^T A W$, Let S denotes the transformed matrix:

$$S = W^T A W \dots\dots\dots(14)$$

Using the properties of inverse matrix, it can retrieve our original matrix:

$$A = (W^T)^{-1} S W^{-1} = (W^{-1})^T S W^{-1} \dots\dots(15)$$

This allows us to see the original image (decompressing the compressed image). The point of doing Haar wavelet transform is that the areas of the original matrix that contain little variation will end up as zero elements in the transformed matrix. A matrix is considered as sparse if it has a “high proportion of zero entries”. Sparse matrices take much less memory to store. Since the transformed matrices are expect to be not always sparse, we decide on a non-negative threshold value known as ϵ , and then let any entry in the transformed matrix whose absolute value is less than ϵ to be reset to zero, this will give us results with a kind of sparse matrix.

Recall that an (n x n) square matrix A is called orthogonal if it's inverse is equal to its transpose. That

latter property makes retrieving the transformed image via the equation (15) much faster.

$$A = (W^T)^{-1} S W^{-1} = (W^{-1})^T S W^{-1} = W S W^T \dots (16)$$

Another powerful property of orthogonal matrices is that they preserve magnitude. In other words, if v is any vector and A is its orthogonal matrix, then $\|Av\| = \|v\|$. Here is how it works:

$$\begin{aligned} \|Av\|^2 &= (Av)^T (Av) = v^T A^T A v \\ &= v^T I v = v^T v = \|v\|^2 \end{aligned} \dots (17)$$

This in turns shows that $\|Av\| = \|v\|$. Also, the angle is preserved when the transformation is done by orthogonal matrices; recall that the cosine of the angle between two vectors u and v is given by:

$$\cos \phi = \frac{u \cdot v}{\|u\| \|v\|} \dots (18)$$

So, if A is an orthogonal matrix, Ψ is the angle between the two vectors Au and Av , then:

$$\begin{aligned} \cos \Psi &= \frac{(Au) \cdot (Av)}{\|Au\| \|Av\|} = \frac{(Au)^T (Av)}{\|u\| \|v\|} = \frac{u^T A^T A v}{\|u\| \|v\|} \\ &= \frac{u^T v}{\|u\| \|v\|} = \cos \phi \end{aligned} \dots (19)$$

Since both magnitude and angle are preserved, there is a significantly less distortion produced in rebuilt image when an orthogonal matrix is used. Since the transformation matrix W is the product of three other matrices, one can normalize W by normalizing each of the three matrices. The normalized version of W is

$$W = \begin{bmatrix} \sqrt{8}/64 & \sqrt{8}/64 & 1/2 & 0 & \sqrt{2}/4 & 0 & 0 & 0 \\ \sqrt{8}/64 & \sqrt{8}/64 & 1/2 & 0 & -\sqrt{2}/4 & 0 & 0 & 0 \\ \sqrt{8}/64 & \sqrt{8}/64 & -1/2 & 0 & 0 & \sqrt{2}/4 & 0 & 0 \\ \sqrt{8}/64 & \sqrt{8}/64 & -1/2 & 0 & 0 & -\sqrt{2}/4 & 0 & 0 \\ \sqrt{8}/64 & -\sqrt{8}/64 & 0 & 1/2 & 0 & 0 & \sqrt{2}/4 & 0 \\ \sqrt{8}/64 & -\sqrt{8}/64 & 0 & 1/2 & 0 & 0 & -\sqrt{2}/4 & 0 \\ \sqrt{8}/64 & -\sqrt{8}/64 & 0 & -1/2 & 0 & 0 & 0 & \sqrt{2}/4 \\ \sqrt{8}/64 & -\sqrt{8}/64 & 0 & -1/2 & 0 & 0 & 0 & -\sqrt{2}/4 \end{bmatrix}$$

For more study, the low and high pass filters can be decomposed to its discrete value to be modified for farther work as [7]:

$$\sum_k h(k) = \sqrt{2} \dots (20)$$

$$\sum_k g(2k) = -\sum_k g(2k+1) \dots (21)$$

For $N=2$, the stability condition enforces:

$$h(0) + h(1) = \sqrt{2} \dots (22)$$

The accuracy condition implies $h(0) - h(1) = 0 \dots (23)$

Thus the solution is:

$$h(0) = \frac{1}{\sqrt{2}}, h(1) = \frac{1}{\sqrt{2}}$$

Modified Haar wavelet transform

To generate Modified Haar Wavelet, it must find the coefficients of $mh(k)$ for low pass filter and $mg(k)$ for high pass filter for different subband N , depending on filter equations (20, 21), First, let $N=3$.

The stability condition for modified Haar enforces:

$$mh(0) + mh(1) + mh(2) = \sqrt{2} \dots (24)$$

The accuracy condition for modified Haar implies:

$$mh(0) - mh(1) + mh(2) = 0 \dots (25)$$

Solving these equations, the different sets of infinitely many solutions (low pass and high pass filter coefficients) obtained for Modified Haar is shown in Table (1):

It is numerically difficult to study and apply all the solutions at the image, as a result it can benefit from the

principle of correlation or energy to get the optimum set where the optimum one is that has minimum energy between others, the normalized energy of any data vector D_i is:

$$E = \frac{1}{N} \sum D_i^2 \dots\dots\dots(26)$$

Where: N is the total number of D_i coefficients.

The energy in wavelet is a good signature that reflects the distribution of coefficient energy along the subsets and has proven to be very powerful for method characteristics.

Table (2) shows the energy distribution .vs. sets.

It is clear that set 4 is of least energy value, so it is the most proposal set that can be used for Modified Haar that providing the optimum PSNR and C.R.

Another solution for obtaining the least error unique solution is by utilizing the powerful MATLAB functions. For our problem the equations (24&25) have more rows than columns and is not of full rank, so matrix A is not a square , then $inv(A)$ does not exist and infinitely solutions are found there. In these cases, “Moore-Penrose pseudo inverse of matrix (A) $pinv(A)$ ” has some of, but not all, the properties of $inv(A)$, the $pinv(A)$ returns the least square errors solution i.e. minimize $norm(A*x-B)$, for the pervious equations the following simple MATLAB routine gives the best solution with minimum error:

```
>> A = [1 1 1;1 -1 1];
>> B = [sqrt(2) 0]';
>> x = pinv(A)*B
```

The best solution is $x= [0.3536 \ 0.7071 \ 0.3536]$ which is compatible with that of minimum energy principle discussed already.

Now, let $N=5$, the equations for the filter coefficients are

The stability condition for modified Haar enforces:

$$mh(0) + mh(1) + mh(2) + mh(4) = \sqrt{2} \quad (27)$$

The accuracy condition for modified Haar implies:

$$mh(0) - mh(1) + mh(2) - mh(4) = 0. \quad (28)$$

Solving these equations, the different sets of infinitely many solutions (low pass and high pass filter coefficients) obtained for modified Haar are shown in Table (3):

The energy test can be used to get the optimum solution constrained for Modified wavelet Haar, Table (4) shows the energy distribution .vs. sets.

It is clear that set 4 is of least energy value, so it is the most proposal set that can be used for Modified Haar through providing the optimum PSNR and C.R, also it can use the MATLAB function “ $pinv(A)$ ” to get the optimum solution as:

```
>> A = [1 1 1 1 1;1 -1 1 -1 1];
>> B = [sqrt(2) 0]';
>> x = pinv(A)*B
```

Results

The flow-diagram of the proposed method is depicted in figure (1) below.

In order to measure and evaluate the performance of our approach, commonly used metric such as the Mean Square Error (MSE) and the Peak Signal to Noise Ratio (PSNR) which are the two error metrics used to compare image compression quality. The MSE represents the cumulative squared error between the compressed and the original image, whereas PSNR represents a measure of the peak error, the lower the value of MSE, the lower the error.

$$MSE = \frac{\sum_{M,N} [I_1(m,n) - I_2(m,n)]^2}{M.N} \quad (29)$$

In the previous equation, M and N are the number of rows and columns in the input images, respectively. Then the block computes the PSNR using the following equation:

$$PSNR = 10 \log_{10} \left(\frac{R^2}{MSE} \right) = 20 \log_{10} \left(\frac{R}{\sqrt{MSE}} \right) \quad (30)$$

In the previous equation, R is the maximum fluctuation in the input image data type. For the color input image it has an 8-bit unsigned integer data type, so R is 255, the higher the PSNR the smaller is the difference between the reconstructed image and the original.

The Haar and proposed Modified Haar methods are compared with JPEG standard compression using three test natural images with 24-bit per pixel color for each and of different size, Figure 2 (a,b,&c) shows typical natural images.

Table (5) shows a comparison between all approaches in bpp (bit per pixel) and PSNR metrics.

Conclusions and suggestions for future works

This paper is aimed to developing computationally efficient algorithm for various image compression using wavelet techniques.

The wavelet based on Haar transform discussed previously is the simplest, and crudest, member of a large class of possibilities, so it can summarized its properties as:

- It is simple, represent as useful tool for image compression, and easy represented in software and even hardware.
- The main benefit of Haar and their Modified types is the sparse representation, fast transformation, low memory space requirements, and possibility of implementation of fast algorithms.
- Due to above points, The Haar transform is preferred where every

time you click on an image to download it from the Internet, the source computer recalls the Haar transformed matrix from its memory. It first sends the overall approximation coefficients and larger detail coefficients and a bit later the smaller detail coefficients. As your computer receives the information, it begins reconstructing in progressively greater detail until the original image is fully reconstructed.

- The powerful of the Modified Haar method is estimated through computing the amount of energy contained in the suggested decomposition coefficients as in table 2 and table 4 for N=3 and N=5.
- As well as, N increased above 2 like N=3 or N=5 (especially odd number) there is an improvement in compression ratio (C.R), and peak signal to noise ratio (PSNR).
- In this work, two types of image compression algorithms are analyzed for many images and the PSNR of around (24-46) dB is obtained for standard Haar and of around (27-50) dB for Modified Haar.
- Haar transform among other transform is feasible for many practical hardware implementations, so image compression can be achieved on DSP chipper as in [4] or on CMOS electronic circuits as in [8].
- It is clear that for low compression ratio C.R (less than 10) as for figure (2a) bird picture shown above, the all techniques are comparable in PSNR and reconstructed images.
- As long as the compression ratio C.R increases as for figure (2c) flowers picture, the distortion in JPEG standard is high, in standard Haar is medium and in modified Haar is low.

- The modified Haar is powerful even though the C.R is large and the PSNR still of satisfactory value.

The data of any image after transformation takes many values, those which are comparable and approach to zero can be tested with threshold value, such a thresholding is appropriate only in the case of coefficients whose absolute values are low. Lowering the level of thresholding may greatly reduce the number of data points required for holding the data at a relatively low quality loss.

For further work, the tradeoff between the value of the threshold and the image quality can be studied and also fixing the correct threshold value can be of great interest. Furthermore, finding out the exact number of transformation levels required in the case of application for a specific image compression can be studied.

References

1. Miheer Gurjar, Prashant Jagannathan, "Image Compression: Review and Comparison of Haar Wavelet Transform and Vector Quantization", 2003.
2. The Internet Encyclopedia, [http://en.Wikipedia.org/wiki/Haar Wavelet](http://en.Wikipedia.org/wiki/Haar_Wavelet).
3. Kamrul Hasan Talukder and Koichi Harada, "Haar Wavelet Based Approach for Image Compression and Quality Assessment of Compressed Image", 2007.
4. Csabo Petre, Vineet Varma, "Wavelet Image Compression on the DSP", 2007.
5. Bushra K. Al-Abudi and Loay A. George, "Color Image Compression Using Wavelet Transform", College of Science, University of Baghdad-Iraq, 2005.
6. Satish Kumar, Kanwarinder Singh, "Image Compression using Discrete Wavelet Transform by Haar Tehnique.
7. Myung-Sin Song, "Wavelet Image Compression", Southern Illinois University Edwardsville, Edwardsville, IL62026, USA, 2004.
8. Ashkan Olyaei, Roman Genov, "Mixed-Signal CMOS Wavelet Compression Imager Architecture", 2005.

Table (1) Solutions of Modified Haar for N=3

Solution no.	Low Pass Coefficients $mh(k)$	High Pass Coefficients $mg(k)$
1	(0.2, 0.7071, 0.5071)	(0.2, -0.7071, 0.5071)
2	(0.261, 0.7071, 0.4461)	(0.261, -0.7071, 0.4461)
3	(0.3, 0.7071, 0.4071)	(0.3, -0.7071, 0.4071)
4	(0.3536, 0.7071, 0.3536)	(0.3536, 0.7071, 0.3536)
5	(0.4, 0.7071, 0.3071)	(0.4, -0.7071, 0.3071)
6	(0.476, 0.7071, 0.2311)	(0.476, -0.7071, 0.2311)
7	Infinity solution	Infinity solution

Table (2) Energy distribution .vs. sets for (N=3) Modified Haar

Set (S_i)	Energy (E_i)
Set 1	0.2657
Set 2	0.2557
Set 3	0.2519
Set 4	0.2500(min.)
Set 5	0.2514
Set 6	0.2599

Table (3) Solutions of Modified Haar for N=5

Soluti on no.	Low Pass Coefficients $mh(k)$	High Pass Coefficients $mg(k)$
1	(0.157, 0.292, 0.25, 0.415, 0.3)	(0.157, -0.292, 0.25, -0.415, 0.3)
2	(0.217, 0.354, 0.215, 0.353, 0.275)	(0.217, -0.354, 0.215, -0.353, 0.275)
3	(0.217, 0.292, 0.215, 0.415, 0.275)	(0.217, -0.292, 0.215, -0.415, 0.275)
4	(0.235, 0.353, 0.235, 0.353, 0.235)	(0.235, -0.353, 0.235, -0.353, 0.235)
5	Infinity solution	Infinity solution

Table (4) energy distribution .vs. sets for (N=5) Modified Haar

Set (S_i)	Energy (E_i)
Set 1	0.0869
Set 2	0.0837
Set 3	0.0852
Set 4	0.083 (min.)

Table (5) techniques comparison

original image	Compression in bpp	PSNR (dB)		
		JPEG	Standard Haar	Modified Haar
Bird	4.8	47	46.2	48.1
Child	0.857	32.6	37	39.8
flowers	0.342	21	24.7	28.9

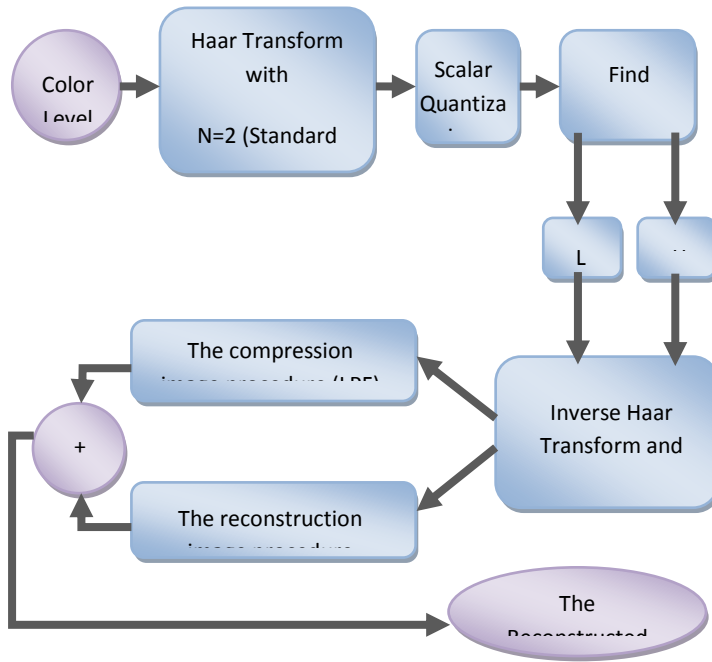


Figure (1) Block diagram of the proposed algorithms



Original Image



Image with JPEG



Image with Haar

Original :Bird
Size:39 KB, 1024×768, 24 bpp



Image with Modified Haar

C.R = 5

Figure (2a), child



Original Image



Image with JPEG



Image with Haar



Image with Modified Haar

Original :Child

Size:77.2 KB, 800×535, 24 bpp

C.R = 28

Figure (2b), child



Original Image



Image with JPEG



Image with Haar



Image with Modified Haar

Original : flowers

Size:465 KB, 1600 ×1200, 24 bpp

C.R=70

Figure (2c), flowers