Variable Length Cyclic Prefix OFDM Using Multipath Delay Tracking

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Abstract

The Cyclic Prefix (CP) technique is widely used in Orthogonal Frequency Division Multiplexing (OFDM) systems. A CP of length greater than the channel order guarantees the orthogonality of the consecutive OFDM symbols. Therefore, the effects of Inter Symbol Interference (ISI) and Inter Carrier Interference (ICI) are cancelled. On the other hand, CP reduces the transmission efficiency.

Conventional OFDM systems use constant length CP. For time varying channels, CP length may become shorter than channel order resulting in considerable loss in orthogonality and performance degradation. Otherwise, when CP length becomes much greater than channel order it results with time and power wasting, and hence decreasing transmission efficiency.

This paper proposes the usage of a multipath delay tracking module to provide the OFDM system with the actual channel order and accordingly the suitable CP length will be determined. Computer simulations show that the proposed system saves about 47.6% of the time and power spent by the same constant length CP OFDM system tested under the same conditions.

Keywords: Adaptive OFDM, Variable CP, Multipath delay tracking, AWGN.

انظمة الاضافة الدورية الإمامية متغيرة الطول باستخدام متعقب التاخر النسبي بين المسارات المتعددة للإشارة OFDM

الخلاصة

تستخدم تقنية الاضافة الدورية بشكل واسع في أنظمة الإرسال متعددة الترددات OFDM. عندما يكون طول مقطع الاضافة الدورية أكبر من رتبة قناة الاتصال، فإنها يضمن صفة التعامدية بين رؤوم البيانات المرسلة. هذه التعامدية تضمن بالتالي امكانية التخلاص من التداخل بين الرؤوم المتعادلة ICI وكذلك التداخل بين الاقتراعات ISI. من الممكن اسخدام الاضافة الدورية بدون تقدير، لكن تستخدم الاضافة الدورية للتخلص من تداخل القناة ISICI، لتحديد رتبة القناة. في هذه الدراسة، تم اقتراح استخدام وحدة تلقيح ومتابعة للقيمة المتغيرة النسبي لل diseñador الزمني النسبي في اسخدام القناة. في هذا الدراسة، تم اقتراح استخدام وحدة تلقيح ومتابعة للقيمة المتغيرة النسبي للデザيزن الزمني النسبي في اسخدام القناة.

الكلمات الدالة: نظام اضافة الدورية متغيرة الطول، متعقب التأخر النسبي للقنوات متعددة المسارات.
Introduction

Linear redundant precoding (LRP) has become popular in digital communication systems due to its capability to facilitate channel equalization of frequency selective time varying channels. By inserting in each transmitted block a redundant segment of a length greater than or equal to the channel order, the inter block interference at the receiver can be eliminated \([1,2]\).

Two major types of LRP techniques are Zero-Padding (ZP) and Cyclic Prefix (CP). ZP systems guarantee symbol recovery regardless of channel null locations, but the CP technique is more widely used in many current standards such as Orthogonal Frequency Division Multiplexing (OFDM)\(^3\). OFDM has been deployed in many applications like IEEE 802.11a, HIPERLAN/2 wireless LANs, Digital Video Broadcasting, and satellite radio.

The OFDM scheme uses symbols generated by a finite length IFFT with size N. The orthogonality of the consecutive OFDM symbols is maintained by appending a length \(\nu\) of CP at the start of each symbol, where \(\nu\) is the length of the Channel Impulse Response (CIR). The CP is obtained by taking the last \(\nu\) samples of each symbol and so the total length of the transmitted OFDM symbol is \((N+\nu)\) samples. By doing this, the linear convolution of the transmitted signal with the CIR is converted into a circular one\(^2\). For each OFDM symbol to be independent and to avoid any Inter Symbol Interference (ISI) and Inter Carrier Interference (ICI), the length of the CIR should be less than \((\nu+1)\) samples. Hence the distortion caused by the CIR only affects the samples within the CP. The receiver takes only the last N samples for decoding at the receiver FFT, discarding the CP. Consequently, the effects of the CIR can then be easily equalized by an array of one-tap frequency domain equalization following demodulation by the FFT \(^3\). Figure 1 shows the block diagram of the system.

One major disadvantage with the OFDM system is the reduction in the transmission efficiency by a factor of \(N/(N+\nu)\) caused by the CP \(^5,6\). This is more serious when the transmitted symbol rate is higher because this makes a CIR with the same relative multipath delay span a greater number of samples hence requiring a longer CP. Moreover, in the case of time varying multipath channels, the length of the CIR varies according to the variation in the dominant multipath delay (D). If a fixed length CP is used (longer than the longest expected CIR length), there will be the possibility that the CP is much longer than the actually required length, leading to wasting in time and transmission power.

One way of increasing the spectrum efficiency is to increase the FFT size, N. However, this increases the complexity of the system and reduces the inter carrier spacing of subcarriers which subsequently makes the system more sensitive to frequency offset and oscillator phase noise \(^6\).

Another way to increase the transmission efficiency is by using variable CP length (\(L_{CP}\)). That is, for a fixed FFT size, the transmission efficiency can be increased by using the shortest \(L_{CP}\) for each transmitted OFDM symbol. This is possible when an accurately enough estimates of \(L_{CP}\) are available for every transmitted OFDM symbol.

Many works have used an OFDM channel estimator at the receiver to
develop information on the channel state. Then, this channel state information is feedback to the transmitter to adapt transmitter parameters like modulation scheme, power allocation, \( L_{CP} \) ...etc \([7,8]\). The operation of adaptive channel estimators depends on the use of training sequences. These training sequences are either transmitted independent of the original data using selected pilot sub-carriers within the OFDM symbol \([8]\), or they are superimposed on the CP \([9,10,11]\).

In either case, the determination of the channel parameters is performed indirectly. That is, the adaptive channel estimator produces an estimate of the CIR from which the parameters of the channel, including \( \nu \), are derived which will in turn used to determine the suitable transmitter parameters like \( L_{CP} \).

Moreover, the performance of these estimators highly depends on the assumption that the received symbols are perfectly detected. That is, incorrect detection of the training symbols will negatively affect the accuracy of the channel estimation and hence the estimated channel parameters. However, the adaptive channel estimators that preserve pilot sub-carriers for the training sequences will reduce the overall system transmission rate.

The disadvantages stated so far can be easily avoided if the required channel parameter, \( \nu \) in this case, is directly derived from the received signal, and then \( L_{CP} \) is calculated at the receiver and feedback to the transmitter. Therefore, this paper proposes the use of a multipath delay tracking system that provides the OFDM system with the actual CIR lengths, according to which the suitable CP length is determined. This paper is organized as follows: in section 2, the proposed system is described, then, computer simulation details while the results are given in section 3. Finally, conclusions are given in section 4.

The Proposed System

Figure (2) shows the block diagram of the proposed system. It consists of an ordinary OFDM system and a multipath delay estimator working in parallel with it to monitor and track the values of \( D \). Once a new estimate for \( \hat{D} \) is produced by the Multipath Delay Tracker (MDT) the corresponding values of \( \nu \) and \( L_{CP} \) will be calculated. The latter is then feedback to the OFDM transmitter.

The MDT has two inputs coming from two sensors with suitable space diversity to receive the signals arriving from the direct and delayed paths independently. These signals are used to adapt an FIR filter using the Least Mean Squares (LMS) algorithm, such that the number of samples due to the relative multipath delay is determined, as shown in Figure 3. An advantage of the MDT over conventional channel estimators is that the MDT dose not requires transmitting specific training sequences and reserving pilot sub-carriers for them. The MDT extracts \( \hat{D} \) from the received signal whether it is ordinary data or CP.

The estimation and tracking processes do not require perfect symbol detection. The details of system adaptation and convergence conditions are given in \([12]\). Then, by assuming that the duration of the direct path is fixed and predetermined, the new length of the total CIR is determined from the estimated value of \( D \) as follows

\[
\text{Channel duration} = \text{direct path duration} + D
\]

\[
\nu T = gT + dT
\]
\[ \nu = g + d \] ..............................(1)

where
\( \nu \) is the number of samples in the total CIR,
\( g \) is the number of samples in the direct path,
\( d \) is the number of delay samples due to \( D \),
\( T \) is the channel sampling time, and
\( \nu, g, \) and \( d \) are integers.

Then
\[ L_{CP} = \left\lceil \nu \right\rceil \] ..............................(2)

where \( \left\lceil \cdot \right\rceil \) refers to the upper integer operator.

**Computer Simulation and Results**

The proposed system shown in Figure (2) is simulated using MATLAB. The communication channel has two dominant skywaves, namely, the direct and delayed paths. The relative time delay between these two paths is variable. The signal at the output of the communication channel is the resultant of the outputs of these two signal paths. Before it enters the OFDM receiver, the received signal is exposed to independent zero-mean Additive White Gaussian Noise (AWGN) samples.

The OFDM system (transmitter and receiver) is simulated for Quadrature Phase Shift Keying (QPSK) data with \( N=128 \) subchannels and a channel sampling rate of \( 1/T = 20000 \) samples per second \( (T=0.05 \text{ ms}) \). Perfect synchronization between the transmitter and receiver is assumed. The duration of an OFDM symbol without extension (CP) can be calculated from the bandwidth of the channel as follows: For a channel bandwidth of 10 kHz, the subchannel width \( \Delta f = 10 \text{ kHz} / 128 = 78.125 \text{ Hz} \). Then, the OFDM symbol duration without CP is \( T_{sym} = 1/\Delta f = 12.8 \text{ ms} \). When CP is added to an OFDM symbol, then the total duration, \( T_{OFDM} \), is given as

\[ T_{OFDM} = T_{sym} + T_{CP} \] ..............................(3)

where \( T_{CP} \) is equal to the number of used CP samples \( (L_{CP}) \) multiplied by \( T \).

Moreover, by assuming four guard subchannels on each end, there are 120 data subchannels each transmitting 2 bits within the time \( T_{OFDM} \). Then, the overall bit rate is calculated as

\[
\text{bit rate} = \frac{2 \text{ bits} \times 120 \text{ subchannels}}{T_{OFDM}} \] ..............................(4)

In the simulations, a total of 10000 OFDM symbols are transmitted. The channel is simulated such that the relative multipath delay changes linearly within intervals of 2000 OFDM symbols. The values of \( D \) at the boundaries of these intervals are given in Table 1, together with the length of the CIR, \( \nu \), calculated using eq.(1) assuming \( g=10 \).

The operation of the MDT is first tested on the channel to evaluate its performance. The two main parameters of interest are the accuracy of the estimates of \( D \) and the maximum number of samples, \( m \), required by the MDT system to settle to a new estimate of \( D \). The former is represented by the mean square error in the values of \( \hat{D} \) with respect to \( D \), as follows

\[
\epsilon = 10 \log_{10} \left| \hat{D} - D \right|^2 \text{ in (dB)} \] ..............................(5)

The MDT is tested for different values of SNR to illustrate the effect of AWGN on its performance, as shown in Figure 4. The values of \( \epsilon \) and \( m \) are given in Table 2 for each tested SNR.
The direct proportionality between the value of SNR and the performance of the MDT system is clear from Table 2. That is the worst MDT performance is expected at lower SNR conditions, where the MDT requires more iterations to settle to a relatively low accuracy estimates of D. However, even for the case when SNR=10 dB in Table 2, the MDT at most requires 53 samples of input data to settle down to a new value of D. That is, the MDT is able to track changes in D many times within the period of one OFDM symbol consisting of at least N samples. This is also true for the other tested values of SNR.

It is important to mention that if \( \hat{D} \) is not an integer multiple of T, it is approximated to the next greater integer; therefore, the value of D is always overestimated by the simulated MDT, leading to CP lengths slightly greater than the actually needed values. This is a good property of the used MDT, which is no underestimates of D will be produced and hence no CP lengths shorter than the actually needed value will be used. This ensures that the CP length suggested by the MDT will always be sufficient to mitigate the effects of ISI and ICI.

As a reference system used for performance comparison, a constant length CP OFDM system is simulated with \( L_{CP} = 76 \) samples to be always greater than the varying values of the CIR length. This reference system is called SYS0.

In order to measure the advantage of using variable length CP OFDM with respect to constant length CP OFDM, the total time required to transmit all of the OFDM symbols is used. That is

\[
T_{tot} = \sum_{i=1}^{10000} T_{OFDMi} = \sum_{i=1}^{10000} T_{sym} + T_{CPi}
\]

\[
= 10000 \times T_{sym} + T \sum_{i=1}^{10000} L_{CPi}
\]

\[
= 128 \text{ (sec)} + 0.05 \text{ (ms)} \times \sum_{i=1}^{10000} L_{CPi}
\]

\[\text{..................(6)}\]

where \( T_{sym} = 12.8 \text{ ms} \) and \( T = 0.05 \text{ ms} \).

Therefore, for SYS0, \( T_{tot} \) is equal to 166 seconds. That is, 166 seconds of time are required by a constant length CP of 76 samples to transmit the 10000 OFDM symbols on the simulated communication channel.

Next, the performance of the proposed variable length CP OFDM system is evaluated. In this work, this system is implemented in two different ways. The resulting systems are called SYS1 and SYS2. Both of these implementations assume that the MDT produces a suggested \( L_{CP} \) continuously once for every OFDM symbol. But they differ from each other in the number of OFDM symbols transmitted before the length of the CP is updated. That is, in SYS1, the \( L_{CP} \) is updated for every OFDM symbol. This scheme ensures the use of the most efficient (shortest) CP length for all of the transmitted OFDM symbols transmitted before the length of the CP is updated. That is, in SYS1, the \( L_{CP} \) is updated for every OFDM symbol. This scheme ensures the use of the most efficient (shortest) CP length for all of the transmitted OFDM symbols. SYS1 is tested for SNR=30 dB. The resulting CP length for each OFDM symbol, \( L_{CPi} \), is plotted in Figure 5. As expected before, \( L_{CPi} \) is slightly greater than \( v \) for all of the transmitted symbols. Therefore, for SYS1, \( T_{CP} \) and \( T_{tot} \) are equal to 19.9 and 147.9 seconds respectively, calculated according to eq.(6). As compared to SYS0 it is clear that SYS1 has reduced the time of
transmitting CP 52.4 % and SYS1 requires only 89.1% of the $T_{tot}$ of SYS0 to transmit the same OFDM symbols and 10.9% of the time and transmitted power is saved. However, the only point in SYS1 that can be considered as a drawback is the heavy traffic load on the feedback control channel. That is this load may exceed its ability to transfer the $L_{CP}$ information properly. Therefore, it is suggested to reduce the feedback traffic by updating $L_{CP}$ regularly once for every $K$ OFDM symbols, where $K$ is an integer. The feedback load is inversely proportional with the value of $K$. The value of $K$ is selected such that an $L_{CP}$ update will still suitable for the next $K$ OFDM symbols before another update is needed. The value of $K$ depends also on the fading rate of the channel, whether it is fast or slow. In the simulations, $K$ is taken to be equal to 20. Meaning that, SYS2 needs to update $L_{CP}$ only 500 times rather than 10000 times in the case of SYS1. But each $L_{CP}$ update in SYS2 must take into account the suitable value of $L_{CP}$ after $K$ OFDM symbols forward from the instant of update. Therefore, $L_{CP}$ is not calculated from the current value of $\hat{D}$ only, but instead, it is calculated from the expected value to occur after the period of $K$ OFDM symbols. This expected value of $\hat{D}$ is extrapolation of the current $\hat{D}$ and its past history values, which must be available (stored) in the MDT. The correctness of the feedback $L_{CP}$ highly depends on the accuracy of the extrapolation process. However, the latter may be completely wrong when $\nu$ decreases unacceptably producing unacceptably long $L_{CP}$, or producing $L_{CP}$ shorter than $\nu$ when it increases unexpectedly. Therefore, SYS2 is further modified by allowing $L_{CP}$ to be updated irregularly according to some criteria, as follows: When the MDT produces an $L_{CP}$ update, then its value is incremented by 1 ($L_{CPx} = L_{CP} + 1$) and feedback such that it will still correct for a number of next OFDM symbols. While the MDT continuously tracks $D$, it calculates $L_{CPi}$ and compares it with the last sent update. If the difference ($L_{CPx} - L_{CP}$) becomes small and/or the slope of $\hat{D}$ values changes sign, then a new $L_{CP}$ update is feedback to the OFDM transmitter, or else the transmitter continues to use $L_{CPx}$, as illustrated in the algorithm shown in Figure 6.

SYS2 is tested for SNR=30 dB. The values of $L_{CP}$ generated and updated by the MDT to be used in the OFDM transmitter are plotted in Figure 7. From this figure, it is clear that due to the proposed algorithm, the actual value of $\nu$ is beneath $L_{CP}$ for all transmitted OFDM symbols leading to a guaranteed ISI and ICI cancellation.

The resulting $T_{CP}$ of SYS2 is 20.6 seconds it is only 45.8% of $T_{CP}$ of SYS0. The total time, $T_{tot}$, for SYS2 is 148.6 seconds. Meaning that SYS2 requires 89.5% of the $T_{tot}$ of SYS0 to transmit the same OFDM symbols and 10.5% of the time and power is saved. But when compared with SYS1, $T_{tot}$ of SYS2 is slightly longer. This is expected since the $L_{CP}$ used is not the exactly correct value for every transmitted OFDM symbol. This is because the number of times $L_{CP}$ is updated is much smaller. In SYS2, $L_{CP}$ is updated 48 times only rather than 10000 times in SYS1. This leads to feedback control channel load reduction to 0.48%, which is the major advantage of SYS2. However, this advantage of SYS2
is at the expense of a slightly more complicated L_CP updating algorithm.

Finally, Table (3) summarizes the relative comparison among the three tested OFDM systems. From Table 3 and the discussion so far, it is concluded that SYS2 seems to be the most suitable to be used in time varying channel environments.

Conclusions

In this work, two implementations of variable length CP OFDM systems operating over a time varying channel are presented. The proposed systems (SYS1 and SYS2) are based on a multipath delay tracking module to produce suitable L_CP updates. SYS1 updates L_CP for every transmitted OFDM symbol whereas SYS2 updates L_CP only when it is necessary to do so. Computer simulation tests show that systems SYS1 and SYS2 can achieve savings of 47.6% and 45.8%, respectively, in time and power required to transmit CP samples with respect to a constant length CP OFDM system tested over the same channel. These results lead to saving in the total time and power required to transmit the 10000 OFDM symbols used in the tests. However, SYS1 suffers from the heavy traffic of L_CP updates that must be feedback before transmitting every OFDM symbol. This problem is solved in SYS2 but at the expense of slightly more complicated L_CP updating algorithm. When compared with conventional adaptive OFDM, the proposed system does not require assigning pilot sub-carriers for training sequences. Therefore, it is concluded that SYS2 seems to be the most suitable tested system to be used in time varying channel environments.

References


Table (1): D and ν variation of the simulated channel

<table>
<thead>
<tr>
<th>OFDM symbol number</th>
<th>D (ms)</th>
<th>ν</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1</td>
<td>12</td>
</tr>
<tr>
<td>2001</td>
<td>0.3</td>
<td>16</td>
</tr>
<tr>
<td>4001</td>
<td>1</td>
<td>30</td>
</tr>
<tr>
<td>6001</td>
<td>1.8</td>
<td>46</td>
</tr>
<tr>
<td>8001</td>
<td>3.3</td>
<td>76</td>
</tr>
<tr>
<td>10000</td>
<td>1.5</td>
<td>40</td>
</tr>
</tbody>
</table>

Table (2): MDT performance

<table>
<thead>
<tr>
<th>SNR (dB)</th>
<th>ε (dB)</th>
<th>m</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>-23.6</td>
<td>53</td>
</tr>
<tr>
<td>15</td>
<td>-45</td>
<td>37</td>
</tr>
<tr>
<td>20</td>
<td>-61.5</td>
<td>22</td>
</tr>
<tr>
<td>25</td>
<td>-67</td>
<td>17</td>
</tr>
<tr>
<td>30</td>
<td>-68.4</td>
<td>13</td>
</tr>
<tr>
<td>40</td>
<td>-68.8</td>
<td>10</td>
</tr>
<tr>
<td>50</td>
<td>-68.8</td>
<td>10</td>
</tr>
</tbody>
</table>

Table (3): Relative comparison among the tested OFDM systems.

<table>
<thead>
<tr>
<th>System</th>
<th>SYS0</th>
<th>SYS1</th>
<th>SYS2</th>
</tr>
</thead>
<tbody>
<tr>
<td>CP Scheme</td>
<td>Constant</td>
<td>Variable</td>
<td>Variable</td>
</tr>
<tr>
<td>T_{CP} (sec)</td>
<td>38</td>
<td>19.9</td>
<td>20.6</td>
</tr>
<tr>
<td>T_{CP} Saving Percentage w.r.t. SYS0</td>
<td>0 %</td>
<td>47.6 %</td>
<td>45.8 %</td>
</tr>
<tr>
<td>T_{tot} (sec)</td>
<td>166</td>
<td>147.9</td>
<td>148.6</td>
</tr>
<tr>
<td>T_{tot} Saving Percentage w.r.t. SYS0</td>
<td>0 %</td>
<td>10.9 %</td>
<td>10.5 %</td>
</tr>
<tr>
<td>Number of L_{CP} updates</td>
<td>——</td>
<td>10000</td>
<td>48</td>
</tr>
</tbody>
</table>

Figure (1) Block diagram of the OFDM system
Figure (2) The proposed system

Figure (3) Multipath delay tracker

Figure (4) MDT performance

Figure (5) $L_{CP}$ of SYS1

\[
\text{L}_{CP} = \text{L}_{CPx} \quad \text{// feedback an initial L}_{CP}
\]

while (NOT End of Transmission) do
begin
Estimate $D$
Calculate $L_{CPi}$ according to eqs. (1) and (2)
If $|L_{CPx} - L_{CP}| \leq 2$ OR slope of $\hat{D}$ changes sign
Update $L_{CP} \quad \text{// feedback a new L}_{CP}$
end

Figure (6) $L_{CP}$ updating algorithm of SYS2

L_{CP} = L_{CPx} \quad \text{// feedback an initial L_{CP}

Figure (7) $L_{CP}$ of SYS2