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Modeling Of Ballistic Missile Dynamics In Pitch Plane And It's Stability

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Abstract

Aerodynamic modeling of ballistic missile in pitch plane is performed and the open-loop transfer function related to the jet deflector angle as input and pitch rate, normal acceleration as output has been derived with certain acceptable assumptions. For typical values of ballistic missile parameters such as mass, velocity, altitude, moment of inertia, thrust, moment and lift coefficient show that, the step time response and frequency response of the missile is unstable. The steady state gain, damping ratio and undraped natural frequency depend on the missile parameters. To stabilize the missile a lead compensator must be added to the forward loop.

Key Words: Missile, Stability, Aerodynamic, Transfer function, Pitch plane

نمذجة ديناميكية الصاروخ البالستيقي في المستوى العمودي واستقراريته

الخلاصة

تم اشتقاق الدوال الانتقالية الايروديناميكية ذات الحلقة المفتوحة للصاروخ البالستيقي في المستوى العمودي حيث كان الادخال زاوية حارفات النفث بينما كان الإخراج معدل تغير زاوية التأرجح والتعجيل العمودي بوجود إفتراضات محددة ومقبولة.

تم اختيار قيم لمعالم الصاروخ مثل الكتلة، السرعة، الارتفاع، عزم القصور الذاتي، قوة الدفع، معاملات العزم ألدوراني ومعاملات قوة الرفع وأظهرت الاستجابة الزمنية والترددية بأن الصاروخ غير مستقر وأن نسبة التخميد والتردد الطبيعي الغير مخمد والكسب يعتمد بشكل رئيسي على معالم الصاروخ. ولغرض استقرارية الصاروخ أثناء الطيران يجب إضافة منظومة سيطرة (معوض سبق) إلى الحلقة الأمامية.

الكلمات الدالة: الصاروخ ، الاستقرارية ، الايروديناميك ، الدالة الانتقالية ، المستوى العمودي.

List of Symbols

a_n	Normal acceleration
B	distance between the center of aerodynamic force application and missile's center of gravity (static stability margin)
$\bar{b}$	distance between the center of gravity and the aerodynamic center of jet deflector
Cg	Center of gravity
$C_{L\alpha}$	lift coefficient due to α
$C_{L\delta}$	lift coefficient due to δ
Cp	Center of pressure
I	moment of inertia
K	Steady state gain
L	Reference length (diameter)
$C_{m\alpha}$	moment coefficient due to α
$C_{m\alpha}$, $C_{m\delta}$, $C_{L\alpha}$, $C_{L\delta}$	stability derivatives
$C_{m\gamma'}$, $C_{m\alpha'}$	damping derivatives
$C_m(\acute{\alpha})$	damping moment coefficient due to $\acute{\alpha}$
$C_{m\delta}$	moment coefficient due to δ
$C_m(\gamma')$	damping moment coefficient due to γ'
D	drag force
G	Gravity
H	Altitude
L (α) , L (δ)	lifting force due to α and δ
m	missile mass
$M_Z(\gamma')$, $M_Z(\acute{\alpha})$	damping moment

M_Z (α), M_Z (γ) aerodynamic moments
q dynamic pressure
S reference area
T missile thrust
V Velocity
W_n Undraped natural frequency
' First derivatives
" Second derivatives

Latin Symbol

α angle of attack
δ Jet deflector angle
Θ flight path angle
γ pitch angle due to α and δ
ζ Damping ratio
ρ air density

Introduction

Normally the complete analysis of missile dynamic required to starts with uncoupled modes of motion (pitch, yaw, and roll) especially in the identification of the transfer function and then study of the stability for each modes, preparing a first step for the controller or compensator designer. The study and analysis of the dynamics in pitch motion as a first step is important stage. The reason is based on the fact that the pitch and yaw are almost identical and also the maneuverability can be significantly determined from the pitch motion.

To obtain the transfer function of the missile it is first necessary to obtain the equations of motion for the missile. The equations of motion are derived by applying Newton's second law of motion which relates the summation of the external forces and moments to the linear and angular accelerations of the system or body. To perform this application, certain assumptions must be made and axis of system must be defined. The assumptions are: first, the center of the axis system is located at the center of gravity of the missile.

The axis system is fixed to the missile and rotates with it, such set of axes are referred as "body axes". Second, the equilibrium or steady-state forces and moments are Zero. Third, the missile is a rigid body. Fourth, the mass of the missile remains constant during any particular dynamic analysis. In order to simplify the calculation where it is assumed that the missile mass is much larger than the fuel mass. Fifth, the missile will be lunched from an earth station. Finally, the earth is an inertial reference.

Many researches analyzed the missile and aircraft stability in pitch plane and simple mathematical model was described.

Titchener [2] discussed the dynamic motion characteristics in pitch plane in relation to the aerodynamic features of the various configurations.

Al-Kindly [4] derived the aerodynamic open-loop transfer function of surface-to-air missile. Jasim [5] derived the aerodynamic open-loop transfer function of the aircraft. In our work the ballistic missile aerodynamic open loop transfer function related to the Jet deflector angle as input and angle of attack , normal acceleration , rate change of missile flight path angle , rate change of missile pitch angle as output has been derived.

Missile Forces Equations

The ballistic missile axis system is shown in figure (1). The complete equation of the force normal to the velocity vector can be written in the following form according to Newton's law.

$$mV\Theta'=L(\alpha)+T(\alpha)-mg\cos(\Theta)+L(\delta) \quad (1)$$

$$mV\Theta'= C_L(\alpha)qS + T\sin(\alpha)-mg\cos(\Theta) + C_L(\delta)qS \quad (2)$$

$$a_n = V\,\Theta' \quad (3)$$

The equation of force in the direction of the velocity vector can be written as:

$$D(\alpha)+ mV' + mg\sin(\Theta) = T\cos(\alpha) \quad (4)$$

$$mV'= T\cos(\alpha)-C_D(\alpha)\, q\, S-mg\sin(\Theta) \quad (5)$$

Missile Rotational Moments Equations

Usually, the missile is designed for stable flight, and any deviation of attack angle from zero gives rise to a moment that tends to resist this deviation. As result, flight of the missile is stable, and its longitudinal axis is continuously oriented in the direction of its velocity vector. The necessary condition for such stable motion is the placement of the center of application of the aerodynamic force behind of the missile's center of gravity. In order to achieve high maneuverability the ballistic missile must be designed statically unstable which means that the placement of the center of pressure a head of the missile's center of gravity. In the above discussed case the pitching moment coefficient due to angle of attack is positive value. To stabilize the unstable missile during the flight a control system must be used, in addition, to the missile. The equation of the moments acting on the missile about its lateral axis Z, which passes through its center of gravity are:

$M_Z (\alpha) = L (\alpha) b$ ……………………... (6)

$MZ (\delta) = L (\delta) \bar{b}$ ………………… (7)

The center of pressure and the center of gravity moves along the longitudinal axis during the flight.

The sum of the external moments is:

$\sum M_Z = M_Z (\alpha) + M_Z (\delta) + M_Z (\gamma') + M_Z (\acute{\alpha})$… (8)

$M_Z = I \gamma''$ ……..……………………….. (9)

$M_Z (\alpha) = q\ S\ L\ C_m (\alpha)$ …………..… (10)

$M_Z (\delta) = q\ S\ L\ C_m (\delta)$ …..……..… (11)

M_Z ('γ) and $M_Z (\acute{\alpha})$ are proportional to the ratio γ '/V and α '/V is

$M_Z (\gamma') = (q / V)\ S\ L2\ C_m (\gamma')$ ..…. (12)

$M_Z (\acute{\alpha}) = (q / V)\ S\ L^2\ C_m (\acute{\alpha})$……. (13)

Mathematical Model of the Missile

The missile force and moment equations are nonlinear because it includes the trigonometric functional Cos(Θ) and Sin(Θ) and the nonlinear depends on the force and moments of attack angle and Jet deflector angle. The input of the mathematical model will be the Jet deflector angle (δ). The normal acceleration (a_n), which is equal to (VΘ'), is taken as output. Equations systems can be rewritten as:

$An = L (\alpha) / m + T \sin (\alpha) / m - g \cos(\Theta) + L (\delta) / m$ …………. (14)

$\Gamma'' = M_Z (\delta) / I + M_Z (\alpha) / I + M_Z (\gamma') / I + M_Z (\acute{\alpha}) / I$ …................................... (15)

$\alpha = \gamma - \Theta$ …………………………. (16)

$\acute{\alpha} = \gamma' - \Theta'$ ……………………….. (17)

Linearization of the Equations of Motion

The first assumption to be adopted consists in excluding the weight term (mg Cos (Θ)) from the equations of motion in the pitch plane, i.e., reducing the equation systems corresponding to the pitch and yaw planes to the same form. This is based on the following physical consideration. Gravity is a systematic, slowly varying force, and hence only the conditions of steady–state motion will differ between the vertical and horizontal planes.

The second assumption is that the angle of attack (α) is small. A direct corollary to this assumption is ($\sin (\alpha) \approx \alpha$).

The following assumption are related to linearization of the aerodynamic lift and moment coefficients $C_L (\alpha, \delta)$, $C_m (\alpha, \delta, \acute{\alpha}, \gamma')$ in a certain range of attack angle α and Jet deflector angle δ these relationships are nearly linear.

They can therefore be approximated as follows in these linear ranges.

$C_L (\alpha) = C_{al.}\ \alpha$ …………………… (18)

$C_L (\delta) = C_{al}\ \delta$ ……..……………… (19)

$C_m (\alpha) = C_{OM}.\ \alpha$ ………………… (20)

$C_m (\delta) = C_{OM}.\ \delta$ …….. ………… (21)

$C_m (\gamma') = C_{OM'}.\ \Gamma'$ ……………… (22)

$C_m (\acute{\alpha}) = C_{OM'}.\alpha'$ ………………… (23)

Taking the above consideration into account the following linear model can be obtained:

$M\,V\,\Theta' = (q\,S\,C_{al} + T)\,\alpha + q\,S\,C_{al}.\,\delta.$ …… (24)

$I'' =$ sly C_{om}l. + slim .α + (q / V) $SL^2 C_{m\gamma'}.\gamma' + (q/V)\,SL^2\,C_{m\alpha'}.\acute{\alpha}$ …. (25)

For simplicity let:

$d_1 = (\rho VS/2m)\,C_{al} + T/mV$... (26)

$d_2 = (\rho V^2\,SL/2I)\,C_{OM}$ ….. (27)

$d_3 = (\rho V^2\,SL/2I)\,C_{OM}$ …… (28)

$d_4 = (vs.\,SL^2/2I)\,C_OM'$ ….. (29)

$d_5 = (\rho VS/2m)\,C_{al}$ ….. (30)

$d_6 = (\rho VSL^2/2I)\,C_{OM'}$ …… (31)

Now equations (26) to (31) can be expressed in the following form:

$\Theta' = d_1\,\alpha + d_5\delta$ ……. (32)

$\Gamma'' = d_2\delta + d_3\,\alpha + d_4\,\gamma' + d_6\acute{\alpha}$... (33)

$\Theta' = \gamma` - \acute{\alpha}$ …… (34)

$\Gamma' - \acute{\alpha} = d_1\,\alpha + d_5\,\delta$ ….. (35)

The equations of force (32), moments (33), and kinematics relationship (34) can be presented in a form of structural diagram as shown in figure (2).

Missile Transfer Function

Taking Laplace transformation on both sides with zero initial condition of equations (32) to (35) we get:

$S\,\gamma(S) - S\alpha(S) = d_1\alpha(S) + d_5\,\delta(S)$... ….. (36)

$S^2\gamma(S) = d_2\,\delta(S) + d_3\,\alpha(S) + S\,d_4\,\gamma(S) + S\,d_6\,\alpha(S)$... …. (37)

The open loop aerodynamic transfer function is:

$\alpha(S)/\delta(S) = (-d_5\,S + d_4\,d_5 + d_2)/(S^2 + (d_1 - d_4 - d_6)\,S - (d_4\,d_1 + d_3))$... ... (38)

$\gamma'(S)/\delta(S) = ((d_2 - d_5 d_6)\,S + d_2 d_1 - d_5 d_3)/(S^2 + (d_1 - d_4 - d_6)\,S - (d_4 d_1 + d_3))$.. (39)

$\Theta'(S)/\delta(S) = \gamma'(S)/\delta(S) - \acute{\alpha}(S)/\delta(S)$

$\Theta'(S)/\delta(S) = d_5\,S^2 - [d_5\,(d_6 + d_4)]\,S + (d_1 d_2 - d_5 d_3)/(S^2 + (d_1 - d_4 - d_6)\,S - (d_4 d_1 + d_3))$... (40)

$a_n(S)/\delta(S) = (V/g) \times \Theta'(S)/\delta(S)$ …. (41)

From equations (38) to (41)

$W_n^2 = -(d_4 d_1 + d_3)$ … (42)

$2\zeta\,W_n = d_1 - d_4 - d_6$ ….. (43)

For steady-state the gain of the system is:

$K = [(d_2 d_1 - d_5 d_3)/-(d_4 d_1 + d_3)] \times (V/g)$ ….. (44)

Results and Discussion

Using the typical values for ballistic missile parameters[1] given in table (1).

The transfer function for a typical values of ballistic missile paremeter which given in table (1) using eqution (41) found as:

$G(S) = a_n(S)/\delta$ (degree) = $(-0.036S^2 - 0.0048S - 0.25)/(S^2 + 0.187S - 0.0\,33)$….. (45)

By using a MATLAB program the time response for unit-step deflection angle[3] was obtained. This response shows that the missile bounded input unbounded output which is unstable as shown in figure (3) and the settling time is none. Also the frequency response (Bode plot) shows that the missile is unstable as shown in figure (4). The gain margin is (31.8 dB) and phase margin is (-160^o). Equations (38) to (41) show that the steady state gain, damping ratio and undamped natural frequency depends on the given parameters mass, moment of inertia, air density, Velocity, Stability derivative and damping derivative.

Conclusions

From the derived transfer function seen that it is second order and the gain of the system varies with the parameters such as, mass of the missile, moment of inertia and Mach number. The undamped natural frequency becomes as a function of the stability derivatives, velocity, air density and missile moment of inertia. The damping ratio varies with the air density, moment of inertia and Mach number. It is deduced that the missile as open-loop transfer function is unstable. To stabilize the missile during the flight and to increase the accuracy of the missile sensing elements such as accelerometer and rate gyroscope must be added to the feedback loop with using lead compensator in the forward loop.

References

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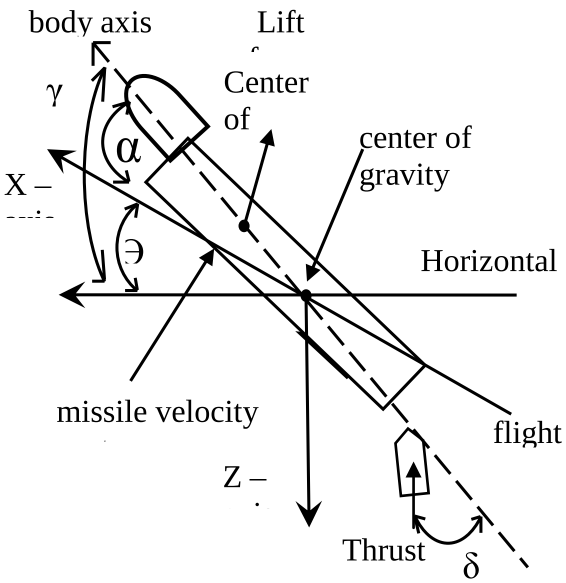


Figure (1): Ballistic Missile

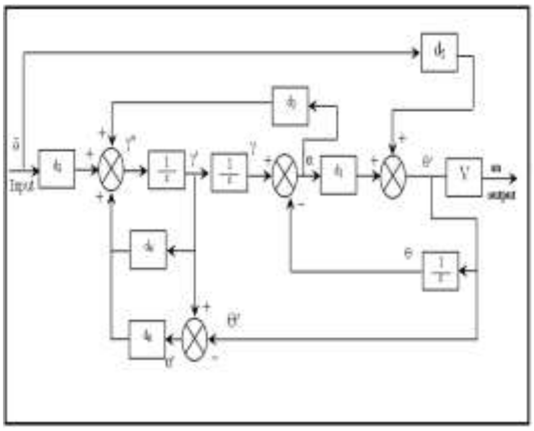
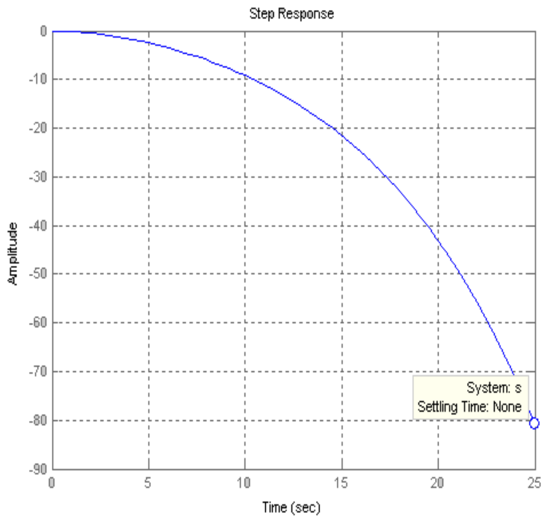


Figure (2): Linearized structural diagram of pitch channel for ballistic missile



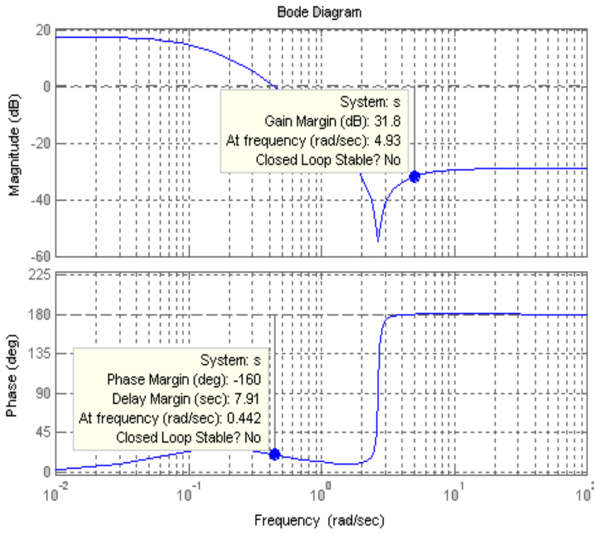


Figure (3): Unit-step

Figure (4): Bode plot

Table (1) Missile Parameters

Symbol	**Value**	**Symbol**	**Value**
T	30722 Ib	$C_{m\delta}$	– 34.25 per radian
V	1285 ft/sec	$C_{m\alpha}$	11.27 per radian (as the Cp is a head of the Cg)
q	585 Ib/sqft	$C_{L\alpha}$	– 3.13 per radian
S	11 soft	$C_{L\delta}$	– 4.63 per radian
L	3.75 ft	C_m (γ')	- 220
I	115000 slug ft^2	C_m (ά)	Zero
H	36000 ft	G	32.18 ft/sec^2
m	445 slugs		